UNIT 1

Electrostatics 8 marks

Chapter 1- Electric charge and field

Electrostatics:

It is the branch of physics which deals with electric charges at rest. It is also known as static electricity. Again it is also called frictional electricity because such charges are produced in insulators on rubbing them with each other.

The charges on insulating bodies cannot move on their own. That is why they are called static charges. Charging by rubbing is due to actual transfer of electrons.

Eg. When we rub a glass rod with silk, electrons are transferred from glass rod to silk. The glass rod becomes positively charged and silk acquires an equal negative charge.

Electric charge:

According to William Gilbert, charges are something possessed by material objects that makes it possible for them to exert electrical force and respond to electrical force.

It is a scalar quality.

[Most common elementary particles are electrons, protons and neutrons.

$m_c = 9.1 \times 10^{-31}$ kg

$m_p = 1.67 \times 10^{-27}$ kg

$m_n = 1.67 \times 10^{-27}$ kg]

Kinds of electric charge:

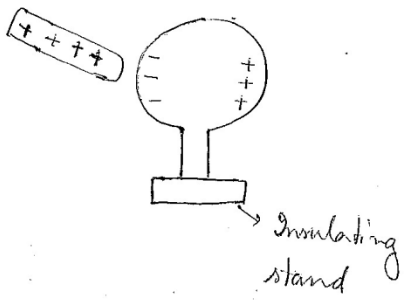
- **i)** Positive charge
- **ii)** Negative charge

Methods of charging or electrification

i) Conductional electrification i.e. charging by physical contract or charging by conduction.
If an uncharged conductor is touched by a charged conductor, the unchanged conductor may acquire charge similar to the charged conductor.

ii) Charging by induction.
A positively charged rod is brought near an uncharged conducting body (placed over an insulated stand). Free electrons of nearer surface of the uncharged body will attract towards the charged rod and the collection of electrons will make it negative.

(-) → bound charge
(+) → free charge



Or,

In charging by induction, a charged body A imparts to another body B, some charge of opposite sign without any actual contact between A and B. (Obviously, body A shall not lose any charge as it is not in contact with B).

Induced charges

The charges appearing on the bodies by the process of induction.

The process by which a charged body makes a conductor charged without physical contract is called charging by instruction.

Basic properties of electric charge.

1. **Additively of electric charge.**
   Electric charge is additive i.e. total charge on an extended body is the algebraic sum of charges in different regions of body.
   Note: Total charge on a body is the sum of all the individual charges i.e. positive and negative charge.
2. **Conservation of electric charge.**
   Accordingly to law of conservation of electric charges, the net electric charge (i.e. algebraic sum of positive and negative charges) in an isolated system remains constant.
3. **Quantization of electric charge.**
   It is the property by virtue of which all free charges are integral multiple of a basic unit of charge represented by e.
   i.e. charge on a body is given by,
   i.e. charge on a body is given by,
   i.e. $q = ne$
   q = total charge
   e = chage on an electron
   Where n is any integer, positive or negative.
   The basic unit of charge is the charge that an electron or proton carrees. By conservation, charge on an electron is taken to be negative. Therefore, charge on an electron is taken as (-e) and charge on proton (+e).

i.e., e = 1.6 X 10^{19} C

What is the cause of quantization of charge ?

Ans: The cause of quantization of charge is that only integral number of electrons can be transferred from one body to another.

COULOMB'S LAW

According to this law, the magnitude of force of interaction between any two point charges at rest is directly proportional to the product of the magnitude of the two charges and inversely proportional to the square of the distance between them.

Or,

The law states that two stationary electric point charges attract or repel each other with a force which is directly proportional to the product of the magnitudes of the charges and inversely proportional to the square of the distance between them.

Let us consider two point charges q_1 and q_2 separated in vacuum by a distance 'r'.

According to Coulomb's law,

$F \alpha\ |q_1|\ |q_2|$

and $F\ \alpha\ \frac{1}{r^2}$

$F\ \alpha\ \frac{|q_1|\ |q_2|}{r^2}$

⇨ $F = k\ \frac{q_1 q_2}{r^2}$--------(1)

Where k is the constant of proportionality and its value depends on the nature of medium separating the charges and on the system of units.

When the charges are situated in free space (air/vacuum), then in cgs system, k = 1.

In SI,

$k=\frac{1}{4\pi\boldsymbol{\epsilon}_{\mathbf{0}}}$

Where $\boldsymbol{\epsilon}_{\mathbf{0}}$ = absolute electrical permittivity of free space.

The, eq^n. (1) becomes,

$F = \frac{1}{4\pi\boldsymbol{\epsilon}_{\mathbf{0}}}\frac{q_1 q_2}{r^2}$, k = 9 X 10^9 Nm^2 C^{-2}

DEFINITION OF ONE COULOMB

In free space,

$F \quad = 9 \times 10^9 \frac{q_1 q_2}{r^2}$

$= \frac{1}{4\pi E_0} \frac{q_1 q_2}{r^2}$

If $q_1 – q_2 = q$, r = 1m

And F = 9×10^9 N, then

$9 \times 10^9 = 9 \times 10^9 \frac{q^2}{1}$

⇨ $q^2 \quad = 1$

q **=** ±1C

Thus, one coulomb is a charge which repels equal and similar charge placed at one meter away in vacuum with a force of 9×10^9 newton.

RELATIVE PERMITTIVITY OR DIELECTRIC CONSTANT

Force between two charges in air (or vacuum) is

$$F_{air} = \frac{1}{4\pi\varepsilon_0} \frac{q_1 q_2}{r^2}$$

Force between the same two charges held same distance apart in a medium of absolute permittivity ε is

$$F_m = \frac{1}{4\pi\varepsilon} \frac{q_1 q_2}{r^2} = \frac{1}{4\pi\varepsilon_0 K} \frac{q_1 q_2}{r^2}$$

$$\therefore \quad \frac{F_{air}}{F_m} = \frac{\varepsilon}{\varepsilon_0} = K = \text{Relative permittivity of the medium}$$

Hence, **relative permittivity** (or **dielectric constant**) *of a medium may be defined as the ratio of force between two charges separated by a certain distance in air (or vacuum) to the force between the same charges separated by the same distance in the medium.*

THE SUPERPOSITION PRINCIPLE:

According to this superposition principle, the total force acting on a given charge is equal to the vector sum of forces exerted on it by all the other charges.

POINT CHARGE:

When the distance between the charged bodies is very large as compared to the sizes of the charged bodies, then the charges are said to be point charges.

CONTINUOUS CHARGE DISTRIBUTION:

A system of closely spaced electric charges forms a continuous charge distribution.

Types:

1. **Linear charge distribution (one dimensional)**

When the charge is uniformly distributed over a line (straight or curved), then we call it **linear charge distribution.**

Eg:- charge on a long wire or a ring.

Let us consider a straight wire AB of length l on which charge q is distributed uniformly.

Let λ = linear charge density of the wire.

$= \frac{\textbf{charge}}{\textbf{length}}$ **Its unit is** *(Cm^{-1})*

2. Surface charge distribution

When the charge is continuously distributed uniformly over a surface, then it is called surface charge distribution.

Eg:- charge on a plane sheet.

Let us consider a plane sheet of surface area S on which charge q is distributed uniformly.

Let σ = surface charge density

$= \frac{\textbf{charge}}{\textbf{area}}$

$= \frac{\textbf{dq}}{\textbf{ds}}$

Its unit is Cm^{-2}.

ELECTRIC FIELD

The **electric field** *due to a charge is the space around the charge in which any other charge experiences a force of attraction or repulsion.*

(i) The charge $+q$ is called the **source charge** because it produces the electric field. The charge $+q_0$ is called the ***test charge.** The test charge should be as small as possible so that its presence does not affect the electric field due to the source charge.

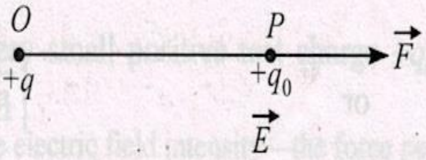
(ii) Theoretically, electric field due to a charge extends up to infinity. However, the effect of electric field dies quickly as the distance from the charge is increased.

ELECTRIC FIELD INTENSITY

The strength of electric field is described by a quantity called *electric field intensity* or *electric field strength*. It is defined as under :

The **electric field intensity** *at a point is defined as the force experienced by unit positive charge placed at that point.*

Consider a point charge $+q$ located at point O in space as shown in Fig. If a small positive test charge $+q_0$ placed at point P experiences a force $\vec{F}$, then electric field intensity $(\vec{E})$ at point P is given by ;

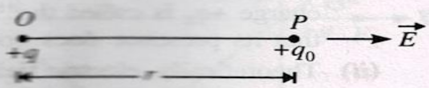


Fig

$$\vec{E} = \frac{\vec{F}}{q_0}$$

Clearly, electric field intensity (= force/charge) is a **vector** *i.e.* it has magnitude and direction. The direction of $\vec{E}$ is that of the force $\vec{F}$ which acts on the positive test charge $+q_0$. The SI unit of electric field intensity is newton per coulomb (N/C).

ELECTRIC FIELD INTENSITY DUE TO A POINT CHARGE



Consider a point charge $+q$ placed at point O. Suppose we are to find electric field intensity at point P distant r from O *i.e.*, $OP = r$. Clearly, point P is in the electric field of $+q$. Imagine a small positive test charge $+q_0$ placed at P. Then force acting on $+q_0$ is

$$\vec{F} = \frac{1}{4\pi\varepsilon_0}\frac{q\,q_0}{r^2}\hat{r}$$

where $\hat{r}$ is a unit vector directed from $+q$ to $+q_0$.

$$\therefore \quad \vec{E} = \frac{\vec{F}}{q_0} = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}\hat{r}$$

or $$|\vec{E}| = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2} = 9\times10^9\,\frac{q}{r^2} \quad ...(i)$$

exp. (*i*) gives the magnitude of electric field intensity arising from charge q at any point at a distance r from $+q$.

ELECTRIC LINES OF FORCE (OR FIELD LINES)

Electric field due to a charge or group of charges is represented by electric lines of force (or field lines). This is a very useful visual representation of electric field. *An electric line of force is the path along which a small positive test charge would move if free to do so.* Following this convention, it is clear that electric lines of force would always originate from a positive charge and end on a negative charge (See Fig). The following relation exists between field lines and electric field.

(*i*) The number of field lines emerging from a positive charge (or terminating on a negative charge) is proportional to the magnitude of the charge.

(*ii*) The field lines point in the direction of the electric field.

(*iii*) The field lines may be straight. *The direction of electric field $\vec{E}$ at any point (such as point P in Fig.) is given by the direction of tangent to the field line at that point.*

(*iv*) The separation of neighbouring field lines indicates the electric field strength in that region. *If the field lines are close together, the electric field in that region is relatively strong; if the field lines are far apart, the field is weak.* Thus in Fig. , near the charge where the force on a test charge would be large, the field lines are close together.

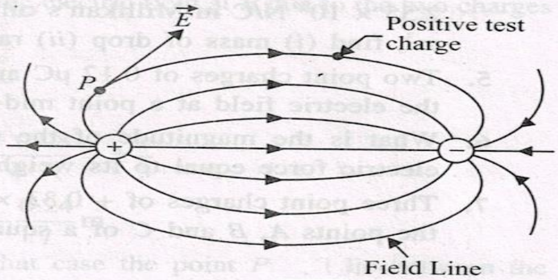


Fig.

Thus representation of electric field by field lines allows us to infer relative field strength as well as direction of the electric field.

PROPERTIES OF ELECTRIC LINES OF FORCE

(*i*) *The electric field lines are directed away from a positive charge and towards a negative charge so that at any point, the tangent to a field line gives the direction of electric field at that point.*

(*ii*) *Electric lines of force start from a positive charge and end on a negative charge.*

(*iii*) **Electric lines of force leave or enter the charged surface normally.**

(*iv*) **Electric lines of force cannot pass through a *conductor. This means that electric field inside a conductor is zero.**

(*v*) **Electric lines of force never intersect each other.** In case the two electric lines of force intersect each other at a point (such as point P in Fig.), then two tangents can be drawn at that point. This would mean two directions of electric field at that point which is impossible.

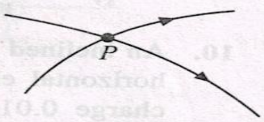


Fig.

(*vi*) *Electric lines of force have tendency to contract in length.* This explains attraction between oppositely charged bodies.

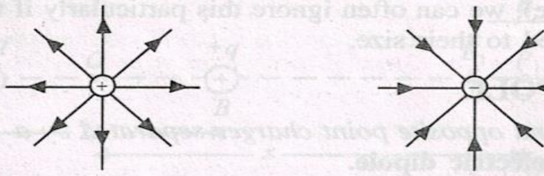
(*vii*) *Electric lines of force have a tendency to expand laterally, i.e.,* they tend to separate from each other in the direction perpendicular to their lengths. This explains repulsion between like charges.

FIELD LINES DUE TO SOME CHARGE CONFIGURATIONS

Electric field lines are an alternative way of representing the electric field due to a charge or group of charges.

(*i*) **Single positive point charge.** Fig. (*i*) shows the electric lines of force (or field lines) due to a single positive point charge. The electric lines of force are directed radially outward. This is expected because a small positive test charge would experience a force directed radially away from the point positive charge.

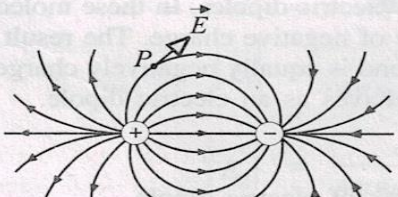
(*ii*) **Single negative point charge.** Fig. (*ii*) shows the electric lines of force due to a single negative point charge. In this case, the electric lines of force are directed radially inward.



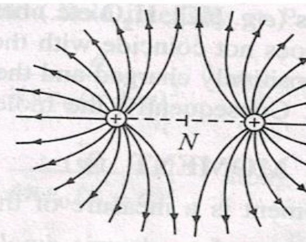
(*i*) (*ii*)

Fig.

(*iii*) **Two equal and opposite point charges.** Fig. shows electric lines of force due to two equal and opposite charges. The electric lines of force are curved in this case and they are directed from the positive charge to the negative charge. Note that direction of the electric field at point *P* is tangent to the field line at point *P*.



(*iv*) **Two equal positive point charges.** Fig. shows electric lines of force due to two equal positive point charges. It is clear that electric lines of force exert lateral pressure which causes repulsion between them. Note that point *N* represents the neutral point where electric field intensity is zero. Since the point charges are of the same magnitude, the point *N* lies at the centre of the line joining the two charges.



ELECTRIC DIPOLE

A system of two equal and opposite point charges separated by a small distance is called an **electric dipole.**

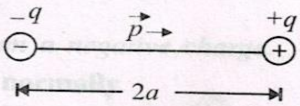


Fig.

Fig. shows an electric dipole consisting of two equal and opposite charges ($-q$, $+q$) separated by a small distance '$2a$'. The line joining the charges is called *dipole axis.* The length of the dipole $(2\vec{a})$ is a *vector* whose direction is from the charge $-q$ to charge $+q$. The total charge of the dipole $= -q + q = 0$. But the electric field due to electric dipole is not zero. It is because the charges $-q$ and $+q$ are separated by some distance and the resultant field due to these charges is not zero.

DIPOLE MOMENT $(\vec{p})$

The dipole moment is a measure of the strength of electric dipole.

The dipole moment of an electric dipole is a vector whose magnitude is equal to the product of either charge and length of the dipole i.e.

$$\vec{p} = q(2a)$$

The direction of $\vec{p}$ is along dipole axis from $-q$ to $+q$. The SI unit of dipole moment *is coulomb-metre* (Cm). The dimensional formula of dipole moment is [M^0 LTA].

Ideal or point dipole. The magnitude of dipole moment of electric dipole is $p = q \times 2a$. If size $2a \to 0$ and charge $q \to \infty$ such that dipole moment remains the same, the resulting dipole is called ideal or point dipole.

A dipole of negligibly small size is called an ideal or point dipole.

DIPOLE FIELD

The electric field produced by an electric dipole is called dipole field.

FIELD INTENSITY ON THE AXIAL LINE OF DIPOLE

Consider an electric dipole consisting of charges $-q$ and $+q$ separated by a small distance $2a$ in free space. Let P be a point on the axial line of the dipole at a distance x from the centre O of the dipole (*i.e.*, $OP = x$). It is desired to find the field intensity at P due to the dipole

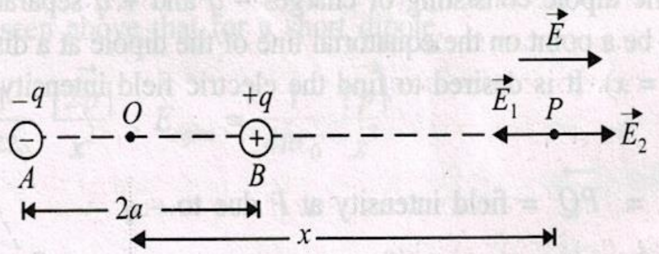


Fig.

Let $\vec{E}_1$ = electric field intensity at P due to $-q$

$\vec{E}_2$ = electric field intensity at P due to $+q$

$\vec{E}$ = resultant electric field intensity at P

$$\left|\vec{E}_1\right| = \frac{q}{4\pi\varepsilon_0 AP^2} = \frac{q}{4\pi\varepsilon_0 (x+a)^2} \quad \text{along } PA$$

$$\left|\vec{E}_2\right| = \frac{q}{4\pi\varepsilon_0 BP^2} = \frac{q}{4\pi\varepsilon_0 (x-a)^2} \quad \text{along } BP \text{ produced}$$

Since $\left|\vec{E_2}\right| > \left|\vec{E_1}\right|$ and they act in opposite directions, resultant field intensity is given by ;

$$\left|\vec{E}\right| = \left|\vec{E_2}\right| - \left|\vec{E_1}\right| \quad \text{along } BP \text{ produced}$$

$$= \frac{q}{4\pi\varepsilon_0}\left[\frac{1}{(x-a)^2} - \frac{1}{(x+a)^2}\right]$$

$$= \frac{q}{4\pi\varepsilon_0}\left[\frac{(x+a)^2 - (x-a)^2}{(x^2-a^2)^2}\right]$$

$$= \frac{q}{4\pi\varepsilon_0}\,\frac{4ax}{(x^2-a^2)^2} = \frac{(q \times 2a) \times 2x}{4\pi\varepsilon_0 (x^2-a^2)^2}$$

$$\therefore \quad \left|\vec{E}\right| = \frac{\left|\vec{P}\right|}{4\pi\varepsilon_0}\,\frac{2x}{(x^2-a^2)^2} \quad \text{along } BP \text{ produced} \qquad \left[\because \left|\vec{p}\right| = q \times 2a\right]$$

If the dipole is short (*i.e.* $2a << x$), then a^2 may be neglected as compared to x^2.

$$\therefore \quad \left|\vec{E}\right| = \frac{1}{4\pi\varepsilon_0}\,\frac{2\left|\vec{p}\right|}{x^3} \quad \text{along } BP \text{ produced}$$

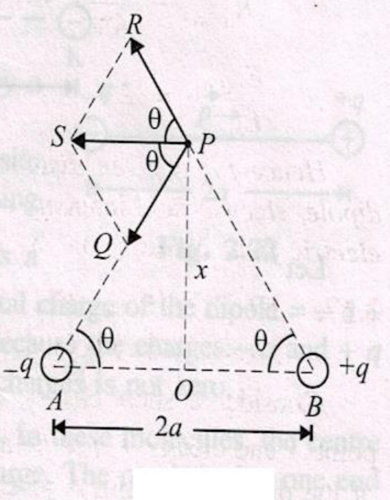
The following points may be noted carefully :

(*i*) The direction of resultant field $\vec{E}$ is along the dipole axis from $-q$ to $+q$.

(*ii*) For a short dipole, $\left|\vec{E}\right| \propto \frac{1}{x^3}$.

FIELD INTENSITY ON EQUATORIAL LINE OF DIPOLE

Consider an electric dipole consisting of charges $-q$ and $+q$ separated by a small distance $2a$ in free space. Let P be a point on the equatorial line of the dipole at a distance x from the centre of the dipole (*i.e.* $OP = x$). It is desired to find the electric field intensity at P due to the dipole



Let $\quad \vec{E_1} = \vec{PQ}$ = field intensity at P due to $-q$

$\quad \vec{E_2} = \vec{PR}$ = field intensity at P due to $+q$

Let $\quad \angle PAB = \angle PBA = \theta$

Since $\quad AP = BP$; $\quad \left|\vec{E_1}\right| = \left|\vec{E_2}\right|$

Now $\quad \left|\vec{E_1}\right| = \left|\vec{E_2}\right| = \frac{1}{4\pi\varepsilon_0}\,\frac{q}{AP^2}$

$$= \frac{1}{4\pi\varepsilon_0}\,\frac{q}{(x^2+a^2)}$$

By parallelogram law of vector additional, total field intensity at P is

$$|\vec{E}| = \sqrt{E_-^2 + E_+^2 + 2E_-E_+ Cos\, 2\theta}$$

$$= \sqrt{2E_-^2 + 2E_-^2 Cos\, 2\theta}$$

$$= \sqrt{2E_-^2 (1 + Cos\, 2\theta})$$

$$= \sqrt{2E_-^2 . 2Cos^2\theta})$$

$$= 2E_- \, Cos\, \theta$$

$$= 2. \frac{1}{4\pi\epsilon_0} \frac{q}{(x^2+a^2)} . \frac{AO}{AP}$$

$$= 2. \frac{1}{4\pi\epsilon_0} \frac{q}{(x^2+a^2)} . \frac{a}{(x^2+a^2)^{1/2}}$$

$$= \frac{q\,(2a)}{4\pi\epsilon_0\,(x^2+a^2)^{3/2}}$$

Therefore,

$$|\vec{E}| = \frac{p}{4\pi\epsilon_0\,(x^2+a^2)^{3/2}} \quad \text{[since p = q(2a)]}$$

Above eqn. gives the expression for magnitude of electric field intensity at a point on the equatorial line of a dipole.

For a short dipole.

x>>2a, $x^2 + a^2 = x^2$

Then,

$$|\vec{E}| = \frac{p}{4\pi\epsilon_0}$$

TORQUE ON DIPOLE IN UNIFORM ELECTRIC FIELD

Consider an electric dipole consisting of charges $-q$ and $+q$ separated by a distance $2a$. Let the dipole be placed in a uniform electric field $\vec{E}$ in such a way that its dipole moment makes an angle θ with the direction of the field as shown in Fig.

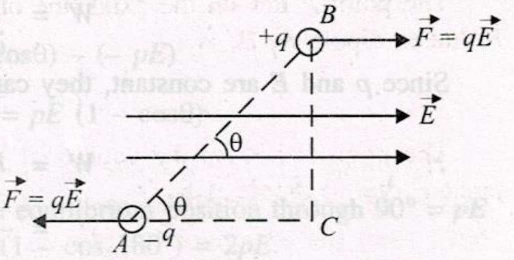


Fig.

Force on charge $+q = q\vec{E}$ along the direction of $\vec{E}$

Force on charge $-q = q\vec{E}$ opposite to the direction of $\vec{E}$.

These two equal and unlike parallel forces constitute a couple which rotates the dipole in clockwise direction, tending to align the dipole along the direction of the field. The magnitude of torque is given by ;

$$\text{Torque, } \tau = \text{Force} \times \text{arm of the couple}$$

$$= F \times BC = (qE)\, 2a \sin\theta$$

$$\therefore \quad \tau = pE \sin\theta \qquad (\because \quad p = q \times 2a)$$

The above expression can be written in vector form as :

$$\vec{\tau} = \vec{p} \times \vec{E}$$

We see that $\vec{\tau}$ is equal to the cross product of $\vec{p}$ and $\vec{E}$. Therefore, $\vec{\tau}$ is perpendicular to the plane containing $\vec{p}$ and $\vec{E}$ and its direction is given by right hand screw rule. The SI unit of torque is Nm and its dimensional formula is $[M L^2 T^{-2}]$.

Special Cases. We discuss the following cases :

(i) **When θ = 0°**, $\tau = pE \sin 0° = 0$

Therefore, torque τ on the dipole is zero when $\vec{p}$ is along $\vec{E}$. In this position, the dipole is in **stable equilibrium.**

(ii) **When θ = 90°**, the torque on the dipole is maximum and is given by ;

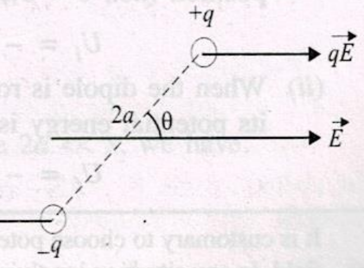
$$\vec{\tau}_{max} = pE \sin 90° = pE$$

Note that when a dipole is placed in a uniform electric field, it experiences a torque only but no net force.

POTENTIAL ENERGY OF DIPOLE IN UNIFORM ELECTRIC FIELD

Consider an electric dipole of dipole moment $\vec{p}$ placed in a uniform electric field $\vec{E}$. Let the dipole moment of the dipole make an angle θ to the direction of electric field Then torque acting on the dipole is given by ;

$$\tau = pE \sin\theta$$



If the dipole is rotated through a very small angle $d\theta$ against this torque, then small amount of work done is

$$dW = \tau\, d\theta = pE \sin\theta\, d\theta$$

∴ Total work done in rotating the dipole from θ_1 to θ_2 is

$$W = \int_{\theta_1}^{\theta_2} pE \sin\theta \, d\theta$$

Since p and E are constant, they can be taken out of the integral.

$$\therefore \quad W = pE \int_{\theta_1}^{\theta_2} \sin\theta \, d\theta = pE[-\cos\theta]_{\theta_1}^{\theta_2}$$

$$= -pE(\cos\theta_2 - \cos\theta_1) \qquad ...(i)$$

Let us suppose that the dipole is initially at *right angles to electric field $\vec{E}$ (*i.e.*, $\theta_1 = 90°$) and is then brought to a position making angle θ with the direction of the field (*i.e.*, $\theta_2 = \theta$). Therefore, eq. (*i*) becomes :

$$W = -pE\cos\theta$$

This work done is stored in the dipole in the form of potential energy (U).

$\therefore$ Potential energy of dipole, $U = -pE\cos\theta$

In vector form, $U = -\vec{p}.\vec{E}$

Clearly, potential energy of an electric dipole is a scalar quantity. It is measured in joule.

Special cases.

(*i*) **When $\theta = 0°$** ; $U = -pE\cos 0° = -pE$

In this position, the dipole has minimum potential energy and hence it is in **stable equilibrium**. The dipole has more potential energy in all other positions.

(*ii*) **When $\theta = 90°$** ; $U = -pE\cos 90° = 0$

In this position, the potential energy of the dipole is zero.

(*iii*) **When $\theta = 180°$** ; $U = -pE\cos 180° = +pE$

In this position, the dipole has maximum potential energy and is in **unstable equilibrium**.

Note. The potential energy of the dipole is minimum when $\vec{p}$ and $\vec{E}$ are in the same direction. However, potential energy of the dipole is maximum when $\vec{p}$ and $\vec{E}$ are in opposite directions.

$$\vec{dS} = \hat{n}\, dS$$

where dS is the magnitude of the area element and $\hat{n}$ is a unit vector in the direction of outward drawn normal to the area element.

Thus **area vector** $\vec{dS}$ *is a vector whose magnitude is the area dS and direction along the outward drawn normal to the area element.*

ELECTRIC FLUX

The **electric flux** *through a given area in an electric field represents the total number of electric lines of force crossing the area.*

Electric flux is a **scalar quantity** and is denoted by the symbol ϕ_E. The electric flux crossing an area is maximum when the area is perpendicular to the electric field.

It is given by,

$d\Phi_E = \vec{E}.\vec{ds}$

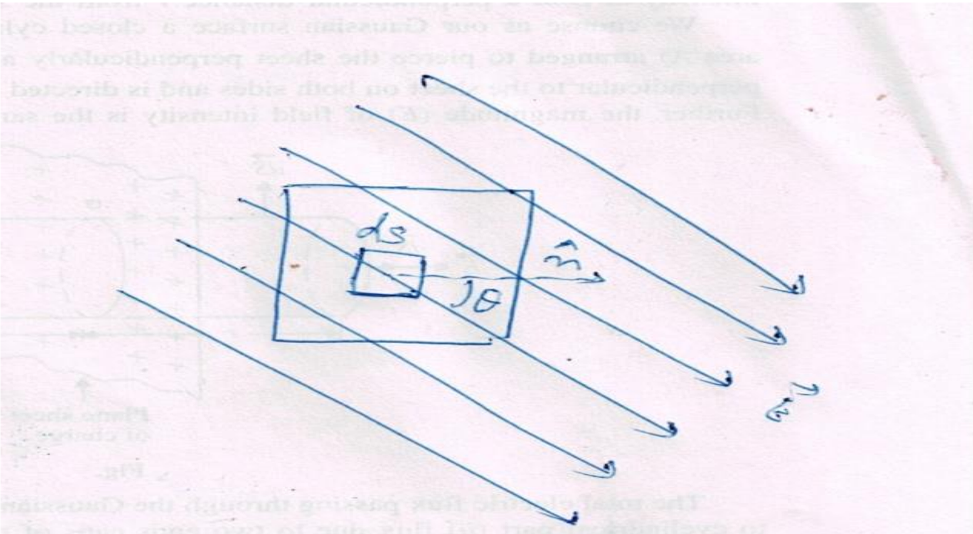
Electric flux over the whole surface is,

$\Phi_E \quad = \int_s \vec{E}.\vec{ds}$

⇨ $\Phi_E \quad = \int_s E.ds$ CosΘ

It is a scalar quantity.

Its S.I. unit is Nm^2c^{-1} or JmC^{-1}



Or,

Electric flux through an area in an electric field may be defined as the total number of electric field lines crossing through that area normally.

$\Phi_E \quad = EA$

In case electric field ($\vec{E}$) is uniform and area S is held normal to the direction of the electric field.

$\Phi_E \quad = ES$

In case the given area is not normal to $\vec{E}$ and $\vec{E}$ makes an angle Θ with normal to the area E, then

$\Phi_E \quad = ES \text{ Cos } \Theta$

If the surface is a closed surface, then the total flux through the closed surface is given by;

$$\phi_E = \oint \vec{E}.\vec{dS} \quad \text{... closed surface}$$

Hence the circle on the integral sign indicates that we are to integrate over a closed surface. *Thus the electric flux through any surface (open or closed) is equal to the surface integral of electric field $\vec{E}$ over that surface.*

Unit of electric flux = Unit of E × Unit of S

∴ SI unit of electric flux = $NC^{-1} \times m^2 = Nm^2\ C^{-1}$

GAUSS'S LAW IN ELECTROSTATICS

Gauss's law or theorem gives relation between the total electric flux (ϕ_E) passing through a closed surface and the net charge (q) enclosed within the surface.

Gauss's law states that the total electric flux passing through a closed surface is equal to $1/\varepsilon_0$ times the net charge enclosed by the closed surface. Here ε_0 is the absolute permittivity of free space (vacuum/air).

If a closed surface encloses a net charge q, then according to Gauss's law, the total electric flux ϕ_E passing through the closed surface is

$$\phi_E = \frac{q}{\varepsilon_0}$$

By definition, $\phi_E = \oint \vec{E}.\vec{dS}$

where $\oint \vec{E}.\vec{dS}$ is the surface integral of electric field $(\vec{E})$ over the entire closed surface enclosing the charge q.

$$\therefore \quad \phi_E = \oint \vec{E}.\vec{dS} = \frac{q}{\varepsilon_0}$$

Hence, Gauss's law may be stated as under :

If a closed surface encloses a net charge (q), then surface integral of electric field $(\vec{E})$ over the closed surface is equal to $1/\varepsilon_0$ times the charge enclosed.

GAUSSIAN SURFACE

A closed surface associated with a charge is called a Gaussian surface.

Or,

The surface chosen to calculate the surface integral.

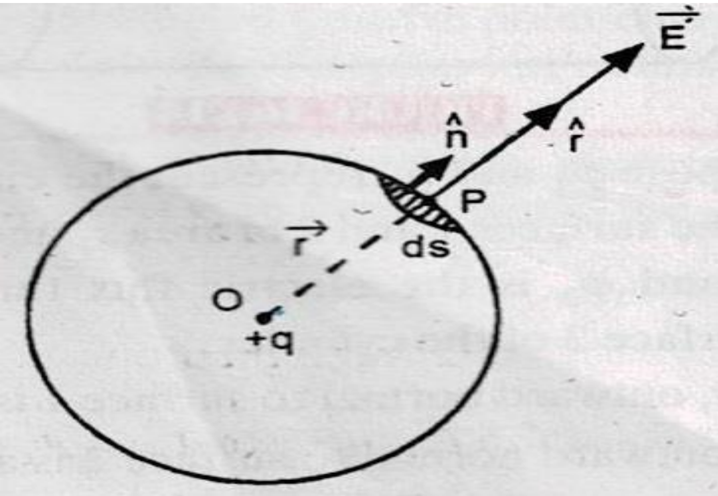
Shape of the Gaussian surface is spherical for a point charge and cylindrical for a line or a sheet charge.

Or,

It is an imaginary closed surface around a point or continuous distribution such that the intensity of electric field at all points on its surface is same.

DEDUCTION OF COULOMB'S LAW FROM GAUSS'S THEOREM/ ELECTRIC FIELD INTENSITY AT ANY POINT DUE TO POINT CHARGE BY USING GAUSS'S THEOREM

Let us consider a charge +q which is placed at origin in a vacuum. Let us calculate the electric field intensity at P due to this charge at a distance r from the charge. A spherical Gaussian surface is drawn passing through the point P.



Let us consider a small area element dS on the Gaussian sphere at P.

By Gauss's theorem

$$\oint_s \vec{E}.\vec{ds} = \frac{q}{\epsilon_0}$$

Or, $\oint_s Eds \cos 0^0 = \frac{q}{\epsilon_0}$

Or, $E\oint_s ds = \frac{q}{\epsilon_0}$

Or, $E(4\pi r^2) = \frac{q}{\epsilon_0}$

$$E = \frac{q}{4\pi\epsilon_0 r^2}$$

This is the electric field intensity at any point P distant r from an isolated point charge q at the centre of the sphere the sphere. If another point charge q_0 were placed at P, then force on q_0 would be

$F = E \times q_0$

$F = \frac{q q_0}{4\pi\epsilon_0 r^2}$

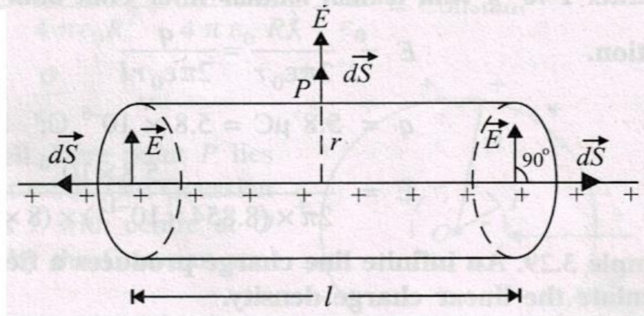
This is Coulomb's Law.

APPLICATIONS OF GAUSS'S LAW

ELECTRIC FIELD INTENSITY DUE TO LINE CHARGE

(Infinitely Long Straight Uniformly Charged wire)

Consider an infinitely long thin wire with uniform linear charge density λ. It is desired to find electric field intensity at any point P at a perpendicular distance r from the axis of the wire



Draw a circular cylinder of radius r and arbitrary length l coaxial with the rod as shown in Fig. . Then this circular cylinder is our Gaussian surface. Since the Gaussian surface must be closed, the two end caps are part of the Gaussian surface. The total electric flux through the Gaussian surface is the algebraic sum of fluxes due to two end caps and cylindrical part of the Gaussian surface.

(i) For all points on the left or right cap, the angle between $\vec{E}$ and area element vector $\vec{dS}$ is 90°. Therefore, the electric flux through the caps is *zero.

$$\oint \vec{E} \cdot \vec{dS} = 0 \quad \text{... for end caps}$$

(ii) The magnitude of $\vec{E}$ on the cylindrical surface is the **same at every point and is directed radially outward from the axis ($\because$ rod is positively charged). Note that angle between $\vec{E}$ and area element vector $\vec{dS}$ is zero. Therefore, only flux is through the curved part of the cylinder and is given by ;

$$\phi_E = \oint \vec{E} \cdot \vec{dS} = \oint E\, dS \cos 0° = E \oint dS$$

or $\qquad \phi_E = E \times †2\pi r l \quad$ ...for cylindrical part

Therefore, electric flux passing through the entire Gaussian surface is $\phi'_E = 0 + \phi_E = 0 + E \times 2\pi rl = E \times 2\pi rl$.

According to Gauss's law, $\phi'_E = \dfrac{q}{\varepsilon_0} = \dfrac{\lambda l}{\varepsilon_0} \qquad (\because q = \lambda l)$

or $\qquad E \times 2\pi r l = \dfrac{\lambda l}{\varepsilon_0}$

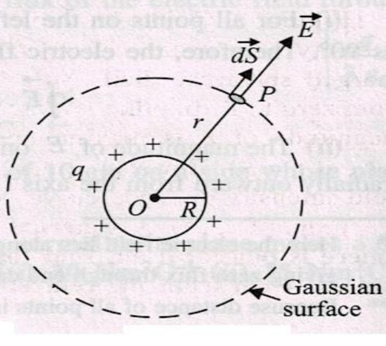
$$\therefore \qquad E = \frac{\lambda}{2\pi\varepsilon_0 r}$$

It is clear that $E \propto 1/r$. Note that direction of $\vec{E}$ is radially outward if the line charge is positive. However, if the line charge is negative, the direction of $\vec{E}$ is radially inward.

ELECTRIC FIELD INTENSITY DUE TO A UNIFORMLY CHARGED SPHERICAL SHELL

Consider a thin spherical shell of radius R and centre O. Let $+q$ be the uniform charge on the surface of the sphere. We are to find electric field intensity due to this charged spherical shell in the following cases :

***(i)* Electric field outside the shell.** We are to find the electric field intensity at any point P outside the shell where $OP = r$. With O as centre, draw a sphere of radius r through point P as shown in Fig. . Then this sphere (dotted) is the Gaussian surface. The electric flux passing through the spherical shell will be the same as that through the Gaussian surface. The



magnitude of electric field (E) is same at all points on the Gaussian surface. Further, at every point on the Gaussian surface, angle between $\vec{E}$ and area element vector $\vec{dS}$ is zero. Therefore, electric flux passing through the Gaussian surface is

$$\phi_E = \oint \vec{E} \cdot \vec{dS} = \oint E\, dS \cos 0° = E \oint dS = E \times 4\pi r^2$$

But according to Gauss's law, $\phi_E = q/\varepsilon_0$.

$$\therefore \quad E \times 4\pi r^2 = \frac{q}{\varepsilon_0}$$

or

$$E = \frac{q}{4\pi \varepsilon_0 r^2}$$

This is the same field intensity which a point charge $+q$ placed at O (centre of the charged sphere) would set up at point P.

Thus electric field intensity at any point outside a uniformly charged spherical shell is the same as if all the charge were concentrated as a point charge at the centre of the shell.

(*ii*) Electric field on the surface of shell. In this case, point P is on the surface of the shell and $r = R$ so that :

$$E = \frac{q}{4\pi\varepsilon_0 R^2}$$

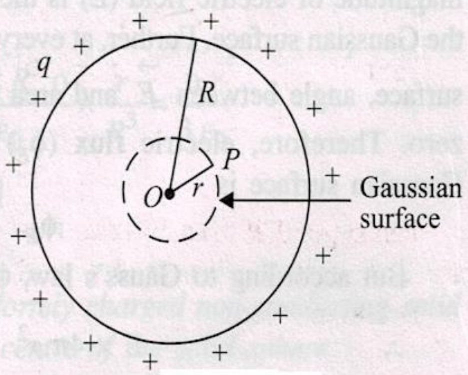
If σ is the surface charge density of the spherical shell, then, $q = 4\pi R^2\sigma$.

$$\therefore \quad E = \frac{q}{4\pi\varepsilon_0 R^2} = \frac{4\pi R^2 \sigma}{4\pi\varepsilon_0 R^2} = \frac{\sigma}{\varepsilon_0} = \text{Constant}$$

i.e.,

$$E = \frac{\sigma}{\varepsilon_0}$$

(*iii*) Electric field inside the shell. Here point P lies inside the shell where $OP = r$. Note that $r < R$. Our Gaussian surface is a sphere (dotted) of radius r with centre at O. The charge enclosed by the Gaussian surface is zero *i.e.* $q = 0$.



Flux through the Gaussian surface is

$$\phi_E = E \times 4\pi r^2$$

According to Gauss's law, we have,

$$\phi_E = \frac{q}{\varepsilon_0} \quad \text{or} \quad E \times 4\pi r^2 = \frac{0}{\varepsilon_0}$$

$$\therefore \quad E = 0 \text{ (for } r < R)$$

Hence the electric field due to a uniformly charged spherical shell is zero at all points inside the shell.

ELECTRIC FIELD INTENSITY DUE TO INFINITE PLANE SHEET OF CHARGE

Consider a thin plane infinite sheet of positive charge having uniform surface charge density σ (charge per unit area) on both sides of the sheet. It is desired to find electric field intensity at P at a perpendicular distance r from the sheet.

We choose as our Gaussian surface a closed cylinder of length $2r$ with end caps (each of area A) arranged to pierce the sheet perpendicularly as shown in Fig. By symmetry, $\vec{E}$ is perpendicular to the sheet on both sides and is directed away from the sheet ($\because$ charge is positive). Further, the magnitude (E) of field intensity is the same over the end caps.

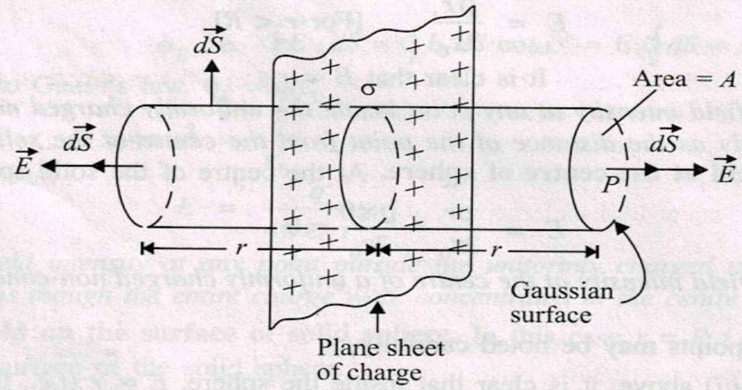


Fig.

The total electric flux passing through the Gaussian surface is the algebraic sum of (*i*) flux due to cyclindrical part (*ii*) flux due to two ends caps of the Gaussian surface.

(*i*) On the cylindrical part of the Gaussian surface, the angle between $\vec{E}$ and area element vector $\vec{dS}$ is 90°. Therefore, there is *no electric flux through the cylindrical part of the Gaussian surface, *i.e.*,

$$\oint \vec{E} \cdot \vec{dS} = 0 \quad \text{... over cylindrical part}$$

(*ii*) At the end caps, angle between $\vec{E}$ and area element vector $\vec{dS}$ is zero. Therefore, only end caps contribute electric flux in our considered Gaussian surface. The electric flux passing through the end caps is given by ;

$$\phi_E = \oint \vec{E} \cdot \vec{dS} + \oint \vec{E} \cdot \vec{dS} = \oint EdS \cos 0° + \oint E\, dS \cos 0°$$

$$= 2E \oint dS = 2EA \qquad (\because \int dS = A)$$

Therefore, total electric flux through the entire Gaussian surface is

$$\phi_E{}' = 0 + \phi_E$$

$$= 0 + 2\,E\,A = 2\,E\,A$$

According to Gauss's law, $\phi_E{}' = q/\varepsilon_0$.

$$\therefore \qquad 2EA = \frac{q^*}{\varepsilon_0} = \frac{\sigma A}{\varepsilon_0} \qquad (\because q = \sigma A)$$

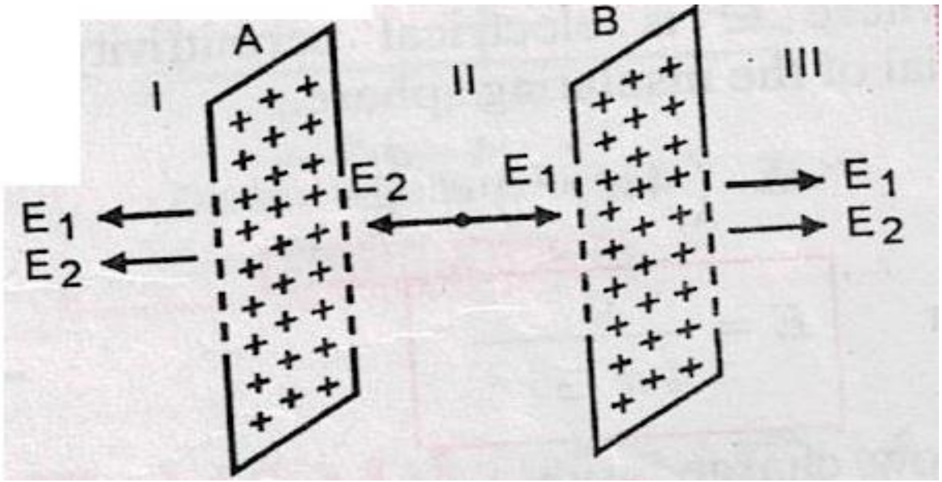
$$\text{or} \qquad E = \frac{\sigma}{2\varepsilon_0}$$

Clearly, E is independent of r, the distance of point P from the plane charged sheet.

If the sheet carries positive charge, the electric field is uniform and points normally away from the plane charged sheet on its both sides. If the sheet carries negative charge, electric field is uniform and points normally into the plane sheet on both sides.

Electric field intensity due to two thin infinite parallel sheets of charge

Let A and B be two thin infinite plane charged sheets held parallel to each other.



Let,

σ_1 = uniform surface charge density of A.

σ_2 = uniform surface charge density of B.

Let a field pointing from left to right is taken as positive and the one pointing from right to left is taken as negative.

∴In region I,

$E_I = -E_1 - E_2$

$\therefore E_I = \frac{-\sigma_1}{2\epsilon_0} - \frac{\sigma_2}{2\epsilon_0} = -\frac{1}{2\epsilon_0}(\sigma_1 + \sigma_2)$ ----------(1)

Similarly, in region II,

$E_{II} = E_1 - E_2$

$= \frac{\sigma_1}{2\epsilon_0} - \frac{\sigma_2}{2\epsilon_0} = \frac{1}{2\epsilon_0}(\sigma_1 - \sigma_2)$ ----------(2)

and in region III,

$E_{III} = E_1 + E_2$

$= \frac{\sigma_1}{2\epsilon_0} + \frac{\sigma_2}{2\epsilon_0} = \frac{1}{2\epsilon_0}(\sigma_1 + \sigma_2)$ ----------(3)

Special cases

If $\sigma_1 = \sigma$, and $\sigma_2 = -\sigma$, i.e. two thin infinite plane sheets with equal and opposite uniform surface densities of charge are held parallel to each other.

From (1), $E_I = 0$

From (3), $E_m = 0$

From (2), $E_{II} = = \frac{2\sigma}{2\epsilon_0} = \frac{\sigma}{\epsilon_0}$ = constant.

Thus field intensity in between such sheets having equal and opposite uniform surface densities of charge becomes constant i.e., a uniform electric field is produced in between two such sheets. Also, E does not depend upon the distance between the thin sheets.