CHAPTER
01

UNIT DIGIT

Last Digit of number is called Unit Digit

1 2 3 4
↓
unit digit

In This no. 4 is unit digit.
The unit digit of the Resultant value depends upon The unit digits of all participating numbers.

Ex.1: 23 + 34 + 46 + 78 = 181, unit digit of 181.

Sol. ∴ unit digit = 1
It is clear that the unit digit of the Resultant value 181 depends upon the unit digits 3, 4, 6, 8
3 + 4 + 6 + 8 = 21
So, units digit = 1

Ex.2: What is the unit digit of ?
31 × 37 × 36 × 46 × 89

sol. 31 × 37 × 36 × 46 × 89
Unit digit = 1, 7, 6, 6, 9
multiply the unit digits = 1 × 7 × 6 × 6 × 9
⇒ 1 × 7 = 7
⇒ 7 × 6 = 42
⇒ 2 × 6 = 12
⇒ 2 × 9 = 18
unit digit = 8

Ex.3: What is the unit digit of?
31 × 33 × 37 × 39 × 43

Sol. 31 × 33 × 37 × 39 × 43
multiply the unit digits
= 1 × 3 × 7 × 9 × 3
unit digit = 7

Ex.4: What is the unit digit of ?
91 × 93 × 95 × 96 × 97 × 98

Sol. multiply the unit digit
1 × 3 × 5 × 6 × 7 × 8 = 0

Ex.5: Find the unit digit of 135 × 136 × 170

Sol. The unit digits = 5, 6, 0
multiply the units digit
= 5 × 6 × 0
= unit digit = 0

Ex.6: Find the unit digit at the product of all the odd prime numbers.

sol. The prime numbers are 3, 5, 7, 11, 13, 17, ... etc.
Now we know that if 5 is multiplied by any odd number it always gives the last digit 5. So the required unit digit will ne '5',

Ex.7: Find the unit digit of 584 × 328 × 547 × 613

Sol. The unit digits = 4, 8, 7, 3
multiplying the unit digits
= 4 × 8 × 7 × 3
= unit digit = 2

Ex.8: Find the unit digit of the product of all the even numbers

Sol. The even number are
2, 4, 6, 8, 10, 12, ... etc.
Now we know that if '0' is multiplied by any number it always gives the last digit 0. so the required unit digit will be 0.

Ex.9: Find the unit digit 4!

Sol. 4! = 4 × 3 × 2 × 1 = 24
unit digit = 4

Ex.10: Find the unit digit 5!

Sol. 5! = 5 × 4 × 3 × 2 × 1 = 120
unit digit = 0

* Factorial 5 and more than 5 express gives unit digit 0.

Unit digit when 'N' is Raised to a power

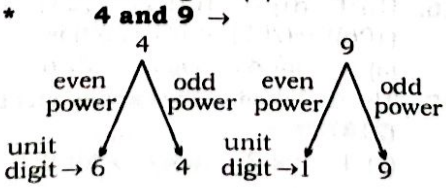
unit digit of 0, 1, 5 and 6 has any power (odd or even) no change

Ex.11: $(3765)^{437}$
unit digit = $(5)^{437} = 5$

Ex.12: $(6736)^{32567}$
unit digit = $(6)^{32567} = 6$

Ex.13: $(32541)^{325}$
unit digit = $(1)^{325} = 1$

* **4 and 9** →



Ex.14: Find the unit place $(67354)^{1237}$

Sol. $(67354)^{1237}$
unit digit = $(4)^{1237} = (4)^{\text{odd power}}$
So, unit digit = 4

Ex.15: Find the unit place $(3259)^{1214}$

Sol. $(3259)^{1214}$
unit digit = $(9)^{1214} = (9)^{\text{even power}}$
unit digit = 1

Ex.16: Find the unit place $(6734)^{312}$

Sol. $(6734)^{312}$
unit digit = $(4)^{312} = (4)^{\text{even}}$
unit digit = 6

Rule of (2, 3, 7 and 8) →
unit digit when 'N' is raised to a power
If the value of the power is

Power →

unit digit ↓	1 or 4n+1	2 or 4n+2	3 or 4n+3	4 or 4n+4
2	2	4	8	6
3	3	9	7	1
7	7	9	3	1
8	8	4	2	6

here n → Natural No.
If those number which unit digit 2, 3, 7 and 8. →
all unit digit have cyclicity 4

Ex.18: Find the unit place 3^{35}

Sol. $3^{35} = 3^{32} \times 3^3$
Break the power form of 4n
$(3^4)^8 \times 3^3$ = (......1) × (...7)
unit place = 1 × 7 = 7

Ex.19: Find the unit place $(127)^{39}$

Sol. $(127)^{39}$
unit place = $(7)^{39}$
= $(7)^{36} \times (7)^3 = (7^4)^9 \times (7)^3$
= (...1) × (....3)
unit place = 1 × 3 = 3

Ex.20: Find the unit place $(678)^{562}$

Sol. $(678)^{562}$
unit digit = $(8)^{562}$
= $(8)^{560} \times (8)^2$
= $(8^4)^{140} \times (8)^2$
= (...6) × (...4)
unit digit = 6 × 4 = 24 = **4**

Ex.21: Find the unit place $(327)^{640}$

Sol. $(327)^{640}$

unit digit = $(7)^{640}$

640 is multiple of 4

then = $(7^4)^{160}$

unit digit = $(1)^{160}$ = **1**

Ex.22: Find the unit digit of $(2137)^{753}$

Sol. $(2137)^{753}$

unit digit = $(7)^{753}$

= $(7)^{752} \times 7^1$

= $(7^4)^{188} \times 7^1$

= $(....1) \times 7$

unit digit = 1×7 = **7**

Ex.23: Find the unit digit of $(13)^{2003}$

Sol. $(13)^{2003}$

unit digit = $(3)^{2003}$

= $3^{2000} \times 3^3$

= $(3^4)^{500} \times 3^3$

= $(....1)^{500} \times 27$

= $1 \times 27 = 27$

unit digit = **7**

Ex.24: Find the unit digit of $(22)^{23}$

Sol. $(22)^{23}$

unit digit = $(2)^{23}$

= $(2)^{20} \times 2^3 = (2^4)^5 \times 8$

= $(....6)^5 \times 8$

unit digit = $6 \times 8 = 48$ = **8**

Ex.25: Find the unit digit of $(37)^{105}$

Sol. $(37)^{105}$

unit digit = $(7)^{105}$

= $(7)^{104} \times 7^1$

= $(7^4)^{26} \times 7^1$

= $(...1)^{26} \times 7$

unit digit = 1×7 = **7**

Ex.26: Find the unit place

$(23)^{21} \times (24)^{22} \times (26)^{23} \times (27)^{24} \times (25)^{25}$

Sol. $(23)^{21} \times (24)^{22} \times (26)^{23} \times (27)^{24} \times (25)^{25}$

unit digit = $(3)^{21} \times (4)^{22} \times (6)^{23} \times (7)^{24} \times (5)^{25}$

Break the power multiple of 4

= $3^{20} \times 3^1 \times 4^{22} \times 6^{23} \times (7^4)^6 \times 5^{25}$

(4^{22} → even power; 6^{23} → same digit; 5^{25} → same digit)

= $3 \times 6 \times 6 \times 1 \times 5$

unit digit = 0

Note:- unit digit = even × 5 = '0'

Ex.27: Find the unit place

$(235)^{215} + (314)^{326} + (6736)^{213} + (3167)^{112}$

unit digit

$(5)^{215} + (4)^{326} + (6)^{213} + (7)^{112}$

($(5)^{215}$ → Same; $(4)^{326}$ → even power; $(6)^{213}$ → Same; $(7)^{112}$ → $(7^4)^{28}$)

= 5 + 6 + 6 + 1

unit digit = 18 = 8

Ex.28: Find the unit place of

$$\frac{12^{55}}{3^{11}} + \frac{8^{48}}{16^{18}}$$

Sol. $\frac{12^{55}}{3^{11}} + \frac{8^{48}}{16^{18}}$

$$= \frac{(3 \times 4)^{55}}{3^{11}} + \frac{(2^3)^{48}}{(2^4)^{18}}$$

$$= \frac{3^{55} \times 4^{55}}{3^{11}} + \frac{2^{144}}{2^{72}}$$

= $3^{44} \times 4^{55} + 2^{72}$

unit digt = $(.....1) \times (......4) +$

= 4 + 6 = 10,

unit digit = 0

EXERCISE

1. Find the unit digit of 584 × 389 × 476 × 786
   (a) 7 (b) 3 (c) 4 (d) 6
2. Find the unit digit of
   641 × 673 × 677 × 679 × 681
   (a) 9 (b) 3 (c) 6 (d) 7
3. Find the unit digit of
   $(5627)^{153} \times (671)^{230}$
   (a) 7 (b) 9 (c) 3 (d) 1
4. Find the unit digit of
   $(3625)^{333} \times (4268)^{645}$
   (a) 6 (b) 3 (c) 4 (d) 0
5. Find the unit digit
   $(3694)^{1793} \times (615)^{317} \times (841)^{941}$
   (a) 5 (b) 3 (c) 4 (d) 0
6. Find the unit digit of
   $(7^{95} - 3^{58})$
   (a) 7 (b) 3 (c) 4 (d) 0
7. Find the unit place of
   $(17)^{1999} + (11)^{1999} - (7)^{1999}$
   (a) 0 (b) 1 (c) 2 (d) 7
8. Find the unit digit of
   $3^6 \times 4^7 \times 6^3 \times 7^4 \times 8^2 \times 9^5$
   (a) 6 (b) 9 (c) 0 (d) 2
9. Find the unit digit of 111! (factorial 111).
   (a) 0 (b) 1 (c) 5 (d) 3
10. The last digit of the number obtained by multiplying the numbers 81 × 82 × 83 × 84 × 85 × 86 × 87 × 88 × 89 will be
    (a) 0 (b) 9 (c) 7 (d) 2
11. Find the units digit of the expression $25^{6251} + 36^{528} + 22^{853}$
    (a) 4 (b) 3 (d) 6 (d) 5
12. Find the units digit of the expression $55^{725} + 73^{5810} + 22^{853}$
    (a) 4 (b) 0 (c) 6 (d) 5
13. Find the units digit of the expression $11^1 + 12^2 + 13^3 + 14^4 + 15^5 + 16^6$.
    (a) 1 (b) 9 (d) 7 (d) 0
14. Find the last digit of the number $1^3 + 2^3 + 3^3 + 4^3 .... + 99^3$.
    (a) 0 (b) 1 (d) 2 (d) 5
15. Unit digit in $(264)^{102}+(264)^{103}$ is:
    (a) 0 (b) 4 (c) 6 (d) 8
16. Unit digit $[(251)^{98}+(21)^{29}-(106)^{100}+(705)^{35}-(16)^4+259]$ is
    (a) 1 (b) 4 (c) 5 (d) 6
17. The unit digit in the expansion of $(2137)^{754}$ is
    (a) 1 (b) 3 (c) 7 (d) 9
18. The digit in unit's place of t[he] product $(2153)^{167}$ is :
    (a) 1 (b) 3 (c) 7 (d) 9
19. The digit in unit's place of t[he] product
    $(2464)^{1793} \times (615)^{317} \times (131)^{491}$ i[s]
    (a) 0 (b) 2 (c) 3 (d) 5
20. What will be the unit digit in t[he] product of 7^{105} ?
    (a) 5 (b) 7 (c) 9 (d) 1
21. what is the number of unit pla[ce] in $(329)^{78}$
    (a) 1 (b) 7 (c) 9 (d) 3
22. unit digit of the number (22
    is:
    (a) 4 (b) 6 (c) 6 (d) 8
23. The unit digit in the produ[ct] $(122)^{173}$ is:
    (a) 2 (b) 4 (c) 6 (d) 8
24. The unit digit in the sum of (124
    + $(124)^{373}$ is :
    (a) 5 (b) 4 (c) 2 (d) 0
25. The last digit of $(1001)^{2008}$ + (1002
    (a) 0 (b) 3 (c) 4 (d) 6

26. Find the unit digit in the product: $(4387)^{245} \times (621)^{72}$.
(a) 1 (b) 2 (c) 5 (d) 7

27. The unit digit of the expression $25^{6251} + 36^{528} + 73^{54}$ is
(a) 6 (b) 5 (c) 4 (d) 0

28. The unit's digit in the product $7^{71} \times 6^{63} \times 3^{65}$ is
(a) 1 (b) 2 (c) 3 (d) 4

29. The last digit of 3^{40} is
(a) 1 (b) 3 (c) 7 (d) 9

30. The digit in unit's place of the number $(1570)^2 + (1571)^2 + (1572)^2 + (1573)^2$ is :
(a) 4 (b) 1 (c) 2 (d) 3

31. The unit digit in 3 × 38 × 537 × 1256 is
(a) 4 (b) 2 (c) 6 (d) 8

32. The unit digit in the product $(2467)^{153} \times (341)^{72}$ is
(a) 1 (b) 3 (c) 7 (d) 9

33. The unit digit in the product $(6732)^{170} \times (6733)^{172} \times (6734)^{174} \times (6736)^{176}$
(a) 1 (b) 3 (c) 4 (d) 5

34. Find the unit digit of the product of all the prime number between 1 and 99999
(a) 9 (b) 7 (c) 0 (d) N.O.T.

35. Find the unit digit of the product of all the elements of the set which consists all the prime numbers greater than 2 but less than 222.
(a) 4 (b) 5 (c) 0 (d) N.O.T.

36. Find the last digit of $222^{888} + 888^{222}$
(a) 2 (b) 6 (c) 0 (d) 8

37. Find the last digit of $32^{32^{32}}$
(a) 4 (b) 8 (c) 6 (d) 2

38. Find the last digit of the expression:
$1^2 + 2^2 + 3^2 + 4^2 + \ldots\ldots + 100^2$.
(a) 0 (b) 4 (c) 6 (d) 8

39. Find the unit digit of $1^1 + 2^2 + 3^3 + \ldots\ldots 10^{10}$.
(a) 9 (b) 7 (c) 0 (d) N.O.T.

40. Find the unit digit of
$13^{24} \times 68^{57} + 24^{13} \times 57^{68} + 1234 + 5678$.
(a) 4 (b) 7 (c) 0 (d) 8

41. The unit digit of the expression
$$\frac{1 \times 2 \times 3 \times 4 \times 5 \times 6 \times 7 \times 8 \times 9 \times 10}{100}$$
(a) 7 (b) 9 (c) 8 (d) N.O.T.

42. Find the unit digit of the expression $888^{9235!} + 222^{9235!} + 666^{2359!} + 999^{9999!}$.
(a) 5 (b) 9 (c) 3
(d) None of these

43. The last digit of the following expression is: $(1!)^1 + (2!)^2 + (3!)^3 + (4!)^4 + \ldots. + (10!)^{10}$
(a) 4 (b) 5 (c) 6 (d) 7

44. The last 5 digits of the following expression will be
$(1!)^5 + (2!)^4 + (3!)^3 + (4!)^2 + (5!)^1 + (10!)^5 + (100!)^4 + (1000!)^3 + (10000!)^2 + (100000!)$
(a) 45939 (b) 00929
(c) 20929
(d) Can't determined

45. The unit digit of the following expression $(1!)^{99} + (2!)^{98} + (3!)^{97} + (4!)^{96} + \ldots\ldots (99!)^1$ is:
(a) 1 (b) 3 (c) 7 (d) 6

46. The unit digit of $(12345k)^{72}$ is 6. The value of k is:
(a) 8 (b) 6 (c) 2
(d) all of these

47. The unit digit of the expression
$(1!)^{1!} + (2!)^{2!} + (3!)^{3!} + \ldots. + (100!)^{100!}$
(a) 0 (b) 1 (c) 2 (d) 7

48. The last digit of the expressioin $4 \times 9^2 \times 4^3 \times 9^4 \times 4^5 \times 9^6 \times \ldots\ldots \times 4^{99} \times 9^{100}$ is :
(a) 4 (b) 6 (c) 9 (d) 1

49. The last digit of the expression $4 + 9^2 + 4^3 + 9^4 + 4^5 + 9^6 + \ldots\ldots + 4^{99} + 9^{100}$ is:
(a) 0 (b) 3 (c) 5
(d) None of these

50. The unit digit of $2^{3^4} \times 3^{4^5} \times 4^{5^6} \times 5^{6^7} \times 6^{7^8} \times 7^{8^9}$ is:
(a) 0 (b) 5
(c) Can't be determined
(d) None of these

ANSWER KEY

1. (d)	6. (c)	11. (b)	16. (b)	21. (a)	26. (d)	31. (d)	36. (c)	41. (c)	46. (d)
2. (a)	7. (b)	12. (c)	17. (d)	22. (d)	27. (d)	32. (c)	37. (c)	42. (b)	47. (d)
3. (a)	8. (a)	13. (b)	18. (c)	23. (a)	28. (d)	33. (c)	38. (a)	43. (d)	48. (b)
4. (d)	9. (a)	14. (a)	19. (a)	24. (d)	29. (a)	34. (c)	39. (b)	44. (b)	49. (a)
5. (d)	10. (a)	15. (a)	20. (b)	25. (b)	30. (a)	35. (b)	40. (a)	45. (c)	50. (a)

SOLUTION

1. (d) 584 × 389 × 476 × 786
unit digit 4, 9, 6, 6
Multiplying the unit digit
= 4 × 9 × 6 × 6
unit digit = 6

2. (a) 641 × 673 × 677 × 679 × 681
unit digit = 1, 3, 7, 9, 1
Multiply the unit digit
= 1 × 3 × 7 × 9 × 1
= 21 × 9 = 189
unit digit = 9

3. (a) $(5627)^{153} \times (671)^{230}$
unit digit $(7)^{153} \times (1)^{230}$
$= (7)^{152} \times 7^1 \times 1$
$= (7^4)^{38} \times 7 \times 1$
$= (\ldots.1)^{38} \times 7$
unit digit = 1 × 7 = 7

4. (d) $(3625)^{333} \times (4268)^{645}$
unit digit $(5)^{333} \times (8)^{645}$
$= 5 \times (8)^{644} \times 8^1$
$= 5 \times (8^4)^{161} \times 8^1$
$= 5 \times (6)^{161} \times 8$
unit digit = 5 × 6 × 8 = 240 = **0**

5. (d) $(3694)^{1793} \times (615)^{317} \times (841)^{941}$
unit digit $(4)^{1793} \times (5)^{317} \times (1)^{941}$
$4^{\text{odd power}} = 4$
$5^n = 5$
4 × 5 × 1 = 20
Hence, unit digit = 0

6. (c) $7^{95} - 3^{58}$

$= 7^{92} \times 7^3 - 3^{56} \times 3^2$

$= (7^4)^{23} \times 343 - (3^4)^{14} \times 9$

$= (.....1)^{23} \times 3 - (.....1)^{14} \times 9$

unit digit = (.....3) – (......9)

$= 13 - 9 = \mathbf{4}$

7. (b) $(17)^{1999} + (11)^{1999} - (7)^{1999}$

unit digit = $(7)^{1999} + (1)^{1999} - (7)^{1999}$

$\because (7)^{1999} - (7)^{1999}$ gives = 0

Then, unit digit = 1

8. (a) Unit digit = $3^6 \times 4^7 \times 6^3 \times 7^4 \times 8^2 \times 9^5$

The unit digit of $3^6 = 3^4 \times 3^2 = 9$

The unit digit of $4^7 = 4$

The unit digit of $6^3 = 6$

The unit digit of $7^4 = 1$

The unit digit of $8^2 = 4$

The unit digit of $9^5 = 9^4 \times 9^1 = 9$

multiply the unit digits = 9 × 4 × 6 × 1 × 4 × 9

unit digit = 6

9. (a) 111! = 1 × 2 × 3 × 4 × 5 × .... × 100 × 111

Since there is product of 5 and 2 hence it will give zero as the unit digit.

Hence the unit digit of 111! is 0 (zero).

10. (a) 81 × 82 × 83 × 84 × 85 × 86 × 87 × 88 × 89

Unit digits = 1, 2, 3, 4, ...,9

Product of unit digits

= 1 × 2 × 3 ×............×9

Because 5 multiply any even no. Then

we gets unit digit = 0

11. (b) $25^{6251} + 36^{528} + 22^{853}$

unit digit = $(5)^{6251} + (6)^{528} + (2)^{853}$

unit digit = $(....5) + (....6) + (2)^{852} \times 2$

$= (....5) + (....6) + (2^4)^{213} \times 2$

$= 5 + 6 + (6)^{213} \times 2$

Sum of unit digit = 5 + 6 + 6 × 2

= 5 + 6 + 12 = 23

Hence, unit digit = 3

12. (c) $55^{725} + 73^{5810} + 22^{853}$

unit digit = $(5)^{725} + (3)^{5810} + (2)^{853}$

$= (...5) + (3^4)^{1452} \times 3^2 + (2^4)^{213} \times 2^1$

$= 5 + (1)^{1452} \times 9 + (16)^{213} \times 2^1$

Sum of unit digit = 5 + 1 × 9 + 6 × 2 = 5 + 9 + 12 = 26

unit digit = 6

13. (b) $11^1 + 12^2 + 13^3 + 14^4 + 15^5 + 16^6$

unit digit = $(1)^1 + (2)^2 + (3)^3 + (4)^4 + (5)^5 + (6)^6$

Sum of unit digit = 1 + 4 + 7 + 6 + 5 + 6 = 29

unit digit = 9

14. (a) $1^3 + 2^3 + 3^3 + 4^3 .... + 99^3$

Sum of cube of natural no.

$$= \left(\frac{n(n+1)}{2}\right)^2 = \left(\frac{99(99+1)}{2}\right)^2$$

$$= \left(\frac{99 \times 100}{2}\right)^2 = (99 \times 50)^2$$

$= (4850)^2$

Unit digit = 0

15. (a) $(264)^{102} + (264)^{103}$

unit digit

$4^1 \rightarrow 4 \rightarrow 4$

$4^2 \rightarrow 16 \rightarrow 6$

$4^3 \rightarrow 64 \rightarrow 4$

Rule: When 4 has odd power, then unit digit is: 4

When 4 has even power, then unit digit is 6

$(264)^{102} + (264)^{103}$

↓ ↓

$(4)^{102} + (4)^{103}$

↓ ↓

(even power) (odd power)

unit digit 6 + 4 = 10 → **0**

Alternate :

$\Rightarrow (264)^{102} + (264)^{103}$

$\Rightarrow (264)^{102} (1 + 264)$

$\Rightarrow (264)^{102} \times 265$

Multiplication of 5 & 2 = 0

Hence, unit digit is 0.

16 (b) $[(251)^{98} + (21)^{29} - (106)^{100} + (705)^{35} - (16)^4 + (259)]$

unit place of 1,5 and 6 will remain same

$= [(1)^{98} + (1)^{29} - (6)^{100} + (5)^{35} - (6)^4 + 9]$

= [1 + 1 – 6 + 5 – 6 + 9]

⇒ 16 – 12 = 4

Hence, unit digit = 4

17. (d) $(2137)^{754}$

$= (7)^{754}$ will give unit digit

	unit digit
$7^1 = 7$	→ 7
$7^2 = 49$	→ 9
$7^3 = 343$	→ 3
$7^4 = 2401$	→ 1
$7^5 = 16807$	→ 7

(cycle of 4) & will repeat

754 divide by 4 = $\frac{75}{4}$

So, remainder i

$7^2 = \mathbf{9}$

Unit Place = 9

18. (c) $(2153)^{167}$

unit digit = 3^{167}

unit digit

$3^1 \rightarrow 3 \rightarrow 3$

$3^2 \rightarrow 9 \rightarrow 9$

$3^3 \rightarrow 27 \rightarrow 7$

$3^4 \rightarrow 81 \rightarrow 1$

This cycle will continue

⇒ divide the power of 3 by 4

$\frac{167}{4}$ ⇒ remainder is 3

$3^3 \Rightarrow 7$

Unit digit = 7

19. (a) $(2464)^{1793} \times (615)^{317} \times (131)$

$4^1 \rightarrow 4 \rightarrow 4$

$4^2 \rightarrow 16 \rightarrow 6$

$4^3 \rightarrow 64 \rightarrow 4$

So odd power of 4 will have 4 as un digit and even power will have 6 unit digit 5 and 1 have same unit di its respectively

$(2464)^{1793} \times (615)^{317} \times (131)^{491}$

↓ odd power ↓

unit digit → 4 × 5 × 1 = 2

⇒ 20 ⇒ 0 unit digit

20. (b) 7^{105}

⇒ $7^1 \rightarrow 7 \rightarrow 7$

⇒ $7^2 \rightarrow 49 \rightarrow 9$

⇒ $7^3 \rightarrow 343 \rightarrow 3$

⇒ $7^4 \rightarrow 2401 \rightarrow 1$

Divide power of 7 by 4

$\frac{105}{4}$ → remainder = 1 ⇒ 7^1 is left

unit digit = 7

21. (a) $(329)^{78}$

⇒ If power of 9 is odd, then unit di number be 9. If power is even th unit digit number be 1.

Hence, unit digit = 1

22. (d) $(22)^{23}$

	Result	unit digit
2^1	2	2
2^2	4	4
2^3	8	8
2^4	16	6
2^5	32	2

Cycle completes

So divide power of 22 by 4

$\frac{23}{4}$ = remainder 3

$2^3 = 8$

unit digit = 8

23. (a) $(122)^{173}$

Unit digit

$2^1 \to 2 \to 2$
$2^2 \to 4 \to 4$
$2^3 \to 8 \to 8$ Cycle
$2^4 \to 16 \to 6$
$2^5 \to 32 \to 2$

$2^{173} = 2^{4\times43+1} = 2^{4\times43}\times2 = 16^{43}\times2$
$= 6^{43}\times2 = 6\times2 = 12$
unit digit = 2

24. (d) $(124)^{372}$ $(124)^{373}$
↓ ↓
4^{372} 4^{373}

When 4 has odd power then unit digit is 4 when 4 has even power then unit digit is 6

$4^1 \to 4 \to 4$
$4^2 \to 16 \to 6$
$4^3 \to 64 \to 4$
$4^4 \to 256 \to 6$

4^{372} 4^{373}
↓ ↓
6 + 4 = 10

last (unit) digit = 0

25. (b) $(1001)^{2008} + 1002$
↓
Unit digit → $1^{2008} + 1002$

Unit digit will be 1 in case of 1 respective of power
⇒ 1+ 1002 = 1003
unit digit (last digit) = 3

26. (d)

unit place

$7^1 \to 7 \to 7$
$7^2 \to 49 \to 9$
$7^3 \to 343 \to 3$ Cycle
$7^4 \to 2401 \to 1$

$(4387)^{245} \times (621)^{72}$
↓
$(7)^{245} \times (1)^{72}$
↓
$(7)^{4\times61+1} \times 1$
↓
$(7)^{4\times61}\times7 \times 1$
↓
$(1)^{61}\times7 \times 1$
unit digit = 7

27. (d) 5 always gives unit digit 5 and 6 always gives unit digit 6

unit digit

$3^1 \to 3 \to 3$
$3^2 \to 9 \to 9$
$3^3 \to 27 \to 7$ Cycle
$3^4 \to 81 \to 1$

$25^{6251} + 36^{528} + 72^{54}$
↓ ↓ ↓
$5^{6251} + 6^{528} + 3^{54}$
↓ ↓ ↓ $54/4$ = r = 2
unit digit → 5 + 6 + 3^2
= 5 + 6 + 9 = 20 = 0
Hence, unit digit = 0

28. (d) $7^{71} \times 6^{63} \times 3^{65}$
↓ ↓ ↓
unit place 7^3 6^3 3^1
↓ ↓ ↓
unit digit ⇒ 3 × 6 × 3 = 54
⇒ 4

29. (a) 3^{40} :

Divide = $\frac{40}{4}$ ⇒ remainder = 0

Unit digit

$3^1 \to 3 \to 3$
$3^2 \to 9 \to 9$ Cycle
$3^3 \to 27 \to 7$
$3^4 \to 81 \to 1$

Hence, unit digit of 3^{40} of completing all cycle = 1

30. (a)

$(1570)^2 + (1571)^2 + (1572)^2 + (1573)^2$
↓ ↓ ↓ ↓
unit digit → $0^2 + 1^2 + 2^2 + 3^2$
↓ ↓ ↓ ↓
0 + 1 + 4 + 9 = 14
unit digit = 4

31. (d) 3 × 3⑧ × 53⑦ × 125⑥
× 2④ × 2⑧ × 4⑧

Note:- Always multiply only unit digit of first no. to second and product's unit digit no. with 3rd no. Again product of last's unit digit to fourth and so on.

Hence, unit digit = 8

32. (c) $(2467)^{153} \times (341)^{72}$
↓ ↓
$(7)^{153} \times (1)^{72}$
↓
[153/4= remainder = 1]

⇒ $7^1 \times 1 = 7$

		Result	Unit digit
7^1	=	7	7
7^2	=	49	9
7^3	=	343	3
7^4	=	2401	1

Hence, unit digit = 7

33. (c) $(6732)^{170} \times (6733)^{172} \times (6734)^{174} \times (6736)^{176}$

unit digit = $(2)^{170} \times (3)^{172} \times (4)^{174} \times (6)^{176}$
$= (2^4)^{42} \times 2^2 \times (3^4)^{43} \times (4)^{174} \times (6)^{176}$
$= (...6) \times 4 \times (...1) \times (...6) \times (...6)$
Multiplication of unit digit
$= 6 \times 4 \times 1 \times 6 \times 6 = 864$
Hence, unit digit = **4**

34. (c) The set of prime number S = {2, 3, 5, 7, 11, 13, ...}

Since there is one 5 and one 2 which gives 10 after multiplying mutually, it means the unit digit will be zero.
Hence, unit digit = 0

35. (b) The set of required prime number = The set of reqired prime number
= {3, 5, 7, 11, ...}

Since there is no any even number is the set so when 5 will multiply with any odd number, it will always give 5 as the last digit.
Hence the unit digit will be 5.

36. (c) The last digit of the expression will be same as the last digit of $2^{888} + 8^{222}$.
Now the last digit of 2^{888} is 6 and the last digit of the 8^{222} is 4.
∴ 6 + 4 = 10.
Hence, unit digit = 0

37. (c) Find the last digit of $2^{32^{32}}$

But $2^{32^{32}} = 2^{32 \times 32 \times 32 \,\ldots\ldots\, \times 32 \text{ times}}$

⇒ $2^{32^{32}} = 2^{4 \times 8 \times (32 \times 32 \,\ldots\ldots\, \times 31 \text{ times})}$

⇒ $2^{32^{32}} = 2^{4n}$

where n = 8 × (32 × 32 .... × 32 times)
Again $2^{4n} = (16)^n$ ⇒ unit digit is 6, for every n ∈ N
Hence, the required unit digit = 6

38. (a) Sum of square natural number = $\frac{n(n+1)(2n+1)}{6}$

Here, n = 100

$= \frac{100\times101\times201}{6} = 338350$

Then, Unit digit = 0

39. (b) Find the unit digit of $1^1 + 2^2 + 3^3 + ...10^{10}$.

The unit digit of $1^1 = 1$
The unit digit of $2^2 = 4$
The unit digit of $3^3 = 7$
The unit digit of $4^4 = 6$
The unit digit of $5^5 = 5$
The unit digit of $6^6 = 6$
The unit digit of $7^7 = 3$
The unit digit of $8^8 = 6$
The unit digit of $9^9 = 9$
The unit digit of $10^{10} = 0$

Thus the unit digit of the given expression will be 7.
($\because$ 1 + 4 + 7 + 6 + 5 + 6 + 3 + 6 + 9 = 47)

40. (a) The unit of 3^{24} is 1
The unit digit of 8^{57} is 8
The unit digit of 4^{13} is 4
The unit digit of 7^{68} is 1
So the resultant value of the unit digits
= 1 × 8 + 4 × 1 + 4 + 8
= 8 + 4 + 4 + 8 = 24
Thus the unit digit of the whole expression is 4.

41. (c) Since in the numerator of the product of the expression there will be 2 zeros at the and thase two zeros will be cancelled by 2 zeros of the denominator. Hence finally we get a non-zero unit digit in the expression.

Now, $\frac{1\times2\times3\times4\times5\times6\times7\times8\times9\times10}{100}$

$= \frac{1\times2^1\times3^1\times2^2\times5^1\times2^1\times3^1\times7^1\times2^3\times3^2\times2^1\times5^1}{5^2\times2^2}$

$= \frac{1\times2^8\times3^4\times5^2\times7^1}{5^2\times2^2}$

$= 1 \times 2^6 \times 3^4 \times 7$

Therefore, the unit digit of the given expression will be same as that of $1 \times 2^6 \times 3^4 \times 7$.

Now, The unit digit of $1 \times 2^6 \times 3^4 \times 7$ is 8.
($\because$ the product of unit digits of 1, 2^6, 3^4, 7 si 1 × 4 × 1 × 7 = 28)

Hence, the unit digit of $\frac{10!}{100}$ is 8.

42. (b) First of all we find the unit digit individually of all the four terms,

So, the unit digit of $888^{9235!}$ is equal to the unit digit of $8^{9235!}$
Now, the unit digit of $8^{9235!}$ equal to the unit of 8^4 (since 9235! is divisible by 4), which is 6.

unit digit of $(888)^{9235!} = (8)^{4n} = 6$
unit digit of $(222)^{9235!} = (2)^{4n} = 6$
unit digit of $(666)^{23591} = (6)^{\text{anypower}} = 6$
unit digit of $(999)^{99991} = (9)^{\text{even power}} = 1$

Thus the unit digit of the expression is 9. ($\because$ 6 + 6 + 6 + 1 = 19)

43. (d) The unit digit of the given expression will be equal to the unit digit of the sum of the unit digits of every term of the expression.

Now, The unit digit of $(1!)^2 = 1$
The unit digit of $(2!)^2 = 4$
The unit digit of $(3!)^3 = 6$
The unit digit of $(4!)^4 = 6$
The unit digit of $(5!)^5 = 0$
The unit digit of $(6!)^6 = 0$
Thus the last digit of the $(7!)^7$, $(8!)^8$, $(9!)^9$, $(10!)^{10}$ will be zero.
So, the unit digit of the given expression = 7
($\because$ 1+4+6 + 6 + 0 + 0 + 0 + 0 + 0 = 17)

44. (b) The last digit of $(1!)^5 = 1$
The last digit of $(2!)^4 = 16$
The last digit of $(3!)^3 = 216$
The last digit of $(4!)^2 = 576$
The last digit of $(10!)^5 = 00000$
The last digit of $(100!)^4 = 00000$
$(1000!)^3 = 00000$
$(10000!)^2 = 00000$
$(100000!)^1 = 00000$
Thus the last 5 digits of the given expression = 00929
[$\because$ 1 + 16 + 216 + 576 + 120 + 00000 + 00000 + 00000 + 00000 + 00000 + = 00929]

45. (c) $(1!)^{99} + (2!)^{98} + (3!)^{97} + (4!)^{96} + ...(99!)^1$
unti digit $(1!)^{99} = (1!)^{99} = 1$
unit digit $(2!)^{98} = 1 \times 2 = (2)^{98} = 4$
unit digit $(3!)^{97} = 1 \times 2 \times 3 = (6)^{97} = 6$
unit digit=$(4!)^{96} = 1 \times 2 \times 3 \times 4 = (4)^{96} = 6$
unit digit=$(5!)^{95} = 1\times2\times3\times4\times 5 = (0)^{95} = 0$
....
....
same unit digit $(99!)^1 = (1\times2....99) = (0)^1 = 0$
Then, Sum of unit digit = 1 + 4 + 6 + 6 + 0 + 0 +.... 0 = 17
unit digit = **7**

46. (d) unit digit $(12345k)^{72} = 6$
if we put the value of k = 2, 6,
Then we get unit digit = 6

47. (d) $(1!)^{1!} + (2!)^{2!} + (3!)^{3!} + ... + (100!)^{100!}$
unit digit $(1!)^{1!} = 1^1 = 1$
unit digit $(2!)^{2!} = (2)^2 = 4$
unit digit $(3!)^{3!} = (6)^6 = 6$
unit digit $(4!)^{4!} = (4)^{24} = 6$
unit digit $(5!)^{5!} = (0)^{120} = 0$
....
....
unit digit $(100!)^{100!}$
$\Rightarrow$ $= (0)^{1\times2\times3\times.......\times100} = 0$
Sum of unit digit = 1 + 4 + 6 + + 0 + 0 + 0 = 17
unit digit = 7

48. (b) $4 \times 9^2 \times 4^3 \times 9^4 \times 4^5 \times 9^6 ...... \times 4^{99} \times 9^{100}$
unit digit $4^1 = 4$
unit digit $9^2 = 1$
unit digit $4^3 = 4$
unit digit $9^4 = 1$
unit digit $4^5 = 4$
....
....
unit digit $4^{99} = 4$
unit digit $9^{100} = 1$
Then multiply the unit digit
4 × 1 × 4 × 1 × 4 × 1 ...... 4 ×
Pair of 4 × 1 (4) is equal 50
we can say this expression = 4
Then, unit place = 6

49. (a) $4 + 9^2 + 4^3 + 9^4 + 4^5 + 9^6 ..... + 4^{99} + 9^{100}$
unit digit $4^1 = 4$
unit digit $9^2 = 1$
unit digit $4^3 = 4$
unit digit $9^4 = 1$
unit digit $4^5 = 4$
....
....
unit digit $4^{99} = 4$
unit digit $9^{100} = 1$
Then,
Sum of unit place
4 + 1 + 4 + 1 + 4+ 1.......4 + 1
Pair of 4 + 1 (5) is equal to 50
We can say this expression
= 50 × 5 = 250
unit digit = 0

50. (a) $2^{3^4} \times 3^{4^5} \times 4^{5^6} \times 5^{6^7} \times 6^{7^8} \times$
We know unit digit of
$2^{3^4} = 2^{81} = 2$
unit digit of $5^{6^7} = 5$
Then we know that when ev number is multplied by 5 th we get unit place = '0'
So, last digit = 0