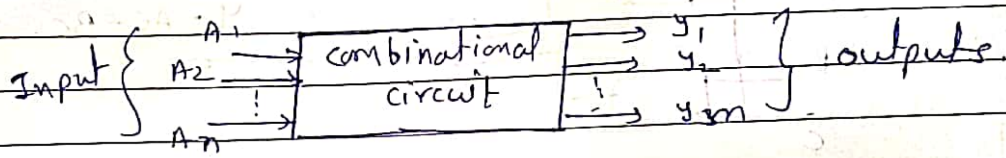
chapter-3
combinational logic circuits

* classification of digital circuits :-
The digital systems in general are classified into two categories namely:
1) combinational logic circuits
2) sequential logic circuits.

combinational logic circuit :-
The output of combinational circuit at any instant of time depents only on the present at input terminals.
The combinational circuits do not use any memory.



Block diagram of combinational ckt.

Examples of combinational circuits.
1) Adders, substractors
2) comparator
3) code converters
4) Encoders, decoders.
5) multiplexers, demultiplexers.

* Methods of simplify the Boolean equations:-
1) Algebric method
2) karnaugh - map (k-map) simplification.
3) Quine - MC cluskey method, and.

4. variable entered mapping (VEM) technique.

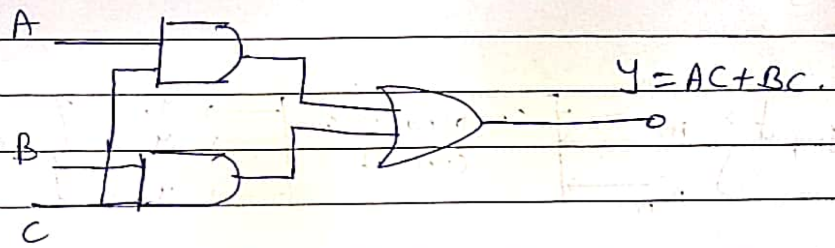
This chapter we will discuss only the karnaugh map or k-map method.

* SOP and POS representation for logic expressions:

consider logic expression given,

$Y = AC + BC.$

Draw in logic gates.



1) sum of products (SOP) form:-

$$Y = \underbrace{AB + AC + BC}_{\text{product terms}}$$

sum

Product terms.

more examples of SOP form:-

$Y = ABC + BCD + AB\bar{D}$

$A = XY + \bar{X}Y + X\bar{Y}$

$Y = \bar{P}\bar{Q} + PQR + PQ\bar{R}$

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2) Product of sums form (POS):-

It is in the form of product of sum terms.

$$Y = (A+\bar{B}+C)\cdot(A+B)\cdot(\bar{A}+C)$$

$$A = (\bar{X}+\bar{Y})\cdot(X+Y+Z)$$

$$Y = (P+R)\cdot(P+\bar{Q})\cdot(\bar{P}+R)$$

Products.

$$Y = (A+B)\cdot(B+C)\cdot(A+C)$$

sum.

* standard or canonical SOP and POS forms.

canonical forms:-

1) canonical SOP form

$$Y = ABC + A\bar{B}\bar{C} + \bar{A}BC$$

each product term consists of all the literals in the complemented or uncomplemented form.

2) canonical POS form:-

$$Y = (A+B+\bar{C})\cdot(A+\bar{B}+C)\cdot(\bar{A}+\bar{B}+\bar{C})$$

each sum term consists of all the literals in the complement or uncomplemented form

* standard form:-

These is another way to express the Boolean functions. It is called as the standard form.

In this form each term may contain one, two or ~~may~~ any number of literals. It is not necessary that each term should contains all the literals.

These can be two types of ~~can~~ standard form

1) standard SOP form.
2) standard POS form.

examples:-

		Type
1.	$Y = AB + AB\bar{C} + \bar{A}BC$	standard SOP
2.	$Y = AB + A\bar{B} + \bar{A}\bar{B}$	canonical SOP
3.	$Y = (\bar{A}+B) \cdot (A+B) \cdot (A+\bar{B})$	canonical POS
4.	$Y = (\bar{A}+B) \cdot (A+B+C)$	standard POS

* conversion of a logic Expression to standard SOP or POS form.

1) conversion from standard SOP to canonical SOP form.

step to be followed:-

step 1:- For each term in the given standard SOP expression find the missing literal.

step 2 :- Then AND this term with the term formed by ORing the missing literal and its compleme

step 3 :- simplify the expression to get the canonical SOP expression.

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example 1:-

1) Convert the expression $Y = AB + A\bar{C} + BC$ into the canonical SOP form.

step 1:- Find missing literal for each term:

$$Y = AB + A\bar{C} + BC$$

missing literal: C, B, A

step 2 :- AND each term with (missing literal + Its complement :

$$Y = \underbrace{AB}_{\text{original product term}} \overset{\text{ANDing}}{\cdot} \underbrace{C(C+\bar{C})}_{\text{missing literal + It's complement}} + A\bar{C}(B+\bar{B}) + BC(A+\bar{A})$$

step3 :- Simplify the expression to get the canonical SOP:-

$$Y = AB(C+\bar{C}) + A\bar{C}(B+\bar{B}) + BC(A+\bar{A})$$

$$= ABC + AB\bar{C} + AB\bar{C} + A\bar{B}\bar{C} + ABC + \bar{A}BC$$

$$= (ABC + ABC) + (AB\bar{C} + AB\bar{C}) + A\bar{B}\bar{C} + \bar{A}BC$$

but $A + A = A$ $\therefore$ $(ABC + ABC) = ABC$

and $(AB\bar{C} + AB\bar{C}) = AB\bar{C}$

$$Y = \underbrace{(ABC + AB\bar{C} + A\bar{B}\bar{C} + \bar{A}BC)}_{\text{each term contains all the literals.}} \leftarrow \text{canonical SOP form.}$$

* conversion from standard pos to canonical pos form.

step 1 :- For each term find the missing literal.

step 2 :- Then OR each term with the term formed by ANDing the missing literal in that term with its complement.

step 3 :- simplify the expression to get the canonical pos.

example :-

1) convert the following expression into canonical SOP form.

1) $\bar{A}+B\bar{C}\bar{D}$ 2) $A\bar{B}C+B\bar{D}$

1) given expression is :-

$$Y=\bar{A}+B\bar{C}\bar{D}$$

$$Y=\bar{A}(B+\bar{B})(C+\bar{C})(D+\bar{D})+B\bar{C}\bar{D}(A+\bar{A})$$

$$=\bar{A}BCD+\bar{A}BC\bar{D}+\bar{A}B\bar{C}D+\bar{A}B\bar{C}\bar{D}+\bar{A}\bar{B}CD$$

$$\cancel{A}\quad+\bar{A}\bar{B}C\bar{D}+\bar{A}\bar{B}\bar{C}D+\bar{A}\bar{B}\bar{C}\bar{D}+$$

$$AB\bar{C}\bar{D}+\bar{A}B\bar{C}\bar{D}$$

$$Y=\bar{A}BCD+\bar{A}BC\bar{D}+\bar{A}B\bar{C}D+\bar{A}B\bar{C}\bar{D}+\bar{A}\bar{B}CD$$

$$+\bar{A}\bar{B}C\bar{D}+\bar{A}\bar{B}\bar{C}D+\bar{A}\bar{B}\bar{C}\bar{D}+AB\bar{C}\bar{D}$$

$\because (A+A=A)$

This is required expression in the standard sop form.

2) The given expression is,

$$Y=A\bar{B}C+B\bar{D}=A\bar{B}C(D+\bar{D})+B\bar{D}(A+\bar{A})(C+\bar{C})$$

$$Y=A\bar{B}CD+A\bar{B}C\bar{D}+ABC\bar{D}+AB\bar{C}\bar{D}+\bar{A}BC\bar{D}+$$

$$\bar{A}B\bar{C}\bar{D}$$

This is required expression in standard sop form.

* concepts of minterm and maxterm :-

minterm :- each individual term in the canonical sop form is called as minterm.

maxterm :- each intividual term in the canonical Pos form is called as maxterm.

canocical sop $Y = ABC + A\bar{B}\bar{C} + \bar{A}B\bar{C}$ → each intivitual term called minterm

canonical pos $Y = (A+B) \cdot (A+\bar{B})$ → each intivitual term called maxterm.

minterm & maxterm for three variables:

variables			minterms	maxterms
A	B	C	m_i	M_i
0	0	0	$\bar{A}\bar{B}\bar{C} = m_0$	$A+B+C = M_0$
0	0	1	$\bar{A}\bar{B}C = m_1$	$A+B+\bar{C} = M_1$
0	1	0	$\bar{A}B\bar{C} = m_2$	$A+\bar{B}+C = M_2$
0	1	1	$\bar{A}BC = m_3$	$A+\bar{B}+\bar{C} = M_3$
1	0	0	$A\bar{B}\bar{C} = m_4$	$\bar{A}+B+C = M_4$
1	0	1	$A\bar{B}C = m_5$	$\bar{A}+B+\bar{C} = M_5$
1	1	0	$AB\bar{C} = m_6$	$\bar{A}+\bar{B}+C = M_6$
1	1	1	$ABC = m_7$	$\bar{A}+\bar{B}+\bar{C} = M_7$

minterm & maxterm for two variables

A	B	m_i	M_i
0	0	$m_0 = \bar{A}\bar{B}$	$M_0 = A+B$
0	1	$m_1 = \bar{A}B$	$M_1 = A+\bar{B}$
1	0	$m_2 = A\bar{B}$	$M_2 = \bar{A}+B$
1	1	$m_3 = AB$	$M_3 = \bar{A}+\bar{B}$

* Representation of logical expression using minterms & maxterms:

1) $Y = \underbrace{ABC}_{m_7} + \underbrace{\bar{A}BC}_{m_3} + \underbrace{A\bar{B}\bar{C}}_{m_4}$ — given logic expression.
— corresponding minterms.

$Y = m_7 + m_3 + m_4 = \Sigma m(3,4,7)$

$\Sigma \rightarrow$ denotes sum of product.
$m \rightarrow$ small m for minterm.

2) $Y = \underbrace{(A+\bar{B}+C)}_{M_2}\ \underbrace{(A+B+C)}_{M_0}\ \underbrace{(\bar{A}+\bar{B}+C)}_{M_6}$

$Y = M_2 M_0 M_6$

$Y = \Pi M(0,2,6)$

$\Pi \rightarrow$ denotes product of sums.
$M \rightarrow$ capital M denotes maxterm.

* examples:-
1) From the truth table obtain the logical expression in the canonical SOP form.

A	B	Y
0	0	0
0	1	1
1	0	1
1	1	0

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→ step 1 → consider only the second and the thired rows of given truth table corresponding to $Y = 1$.

step 2 & 3 :- For second and third entries in table write the product term.

	A	B	Y	
1)	0	0	0	
2)	0	1	1	$\rightarrow Y_1 = \bar{A}B$
3)	1	0	1	$\rightarrow Y_2 = A\bar{B}$
4)	1	1	1	

$$Y = Y_1 + Y_2$$

$$Y = \bar{A}B + A\bar{B}$$

$$Y = m_1 + m_2$$

$$Y = \Sigma m(1, 2)$$

2) example :- 2

For the truth table given write logic expression in the canonical ~~SOP~~ POS form.

A	B	C	Y
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	0
1	0	0	1

A	B	C	Y
1	0	1	0
1	1	0	0
1	1	1	1

→ write ~~product~~ sum term for $Y=0$

1)

A	B	C	Y	
0	0	0	0	$\longrightarrow A+B+C = Y_1$
0	0	1	1	
0	1	0	0	$\longrightarrow A+\bar{B}+C = Y_2$
0	1	1	0	$\longrightarrow A+\bar{B}+\bar{C} = Y_3$
1	0	0	1	
1	0	1	0	$\longrightarrow \bar{A}+B+\bar{C} = Y_4$
1	1	0	0	$\longrightarrow \bar{A}+\bar{B}+C = Y_5$
1	1	1	1	

2) $Y = Y_1 \cdot Y_2 \cdot Y_3 \cdot Y_4 \cdot Y_5$

$Y = (A+B+C)\cdot(A+\bar{B}+C)\cdot(A+\bar{B}+\bar{C})\cdot(\bar{A}+B+\bar{C})\cdot(\bar{A}+\bar{B}+C$

$Y = M_0 \cdot M_2 \cdot M_3 \cdot M_5 \cdot M_6$

$Y = \pi M(0,2,3,4,5)$.

* Method of simplify ~~Boolen~~ Boolean function :-
1) Algebric method
2) karnaugh-map simplification

1) Algebric simplification.

1) example:-
simplify the following three variables Boolean expression.

$$Y = \Sigma m(2,4,6)$$

$$Y = m_2 + m_4 + m_6$$
$$= \bar{A}B\bar{C} + A\bar{B}\bar{C} + AB\bar{C}$$
$$= \bar{A}B\bar{C} + A\bar{C}(\bar{B}+B)$$

but $\bar{B}+B = 1$

$$Y = \bar{A}B\bar{C} + A\bar{C}$$
$$Y = \bar{C}(\bar{A}B + A)$$

but $A+BC = (A+B)(A+C)$

hence,

$$Y = \bar{C}(A+\bar{A})(A+B)$$
$$Y = C(1)(A+B)$$
$$Y = \bar{C}(A+B).$$

2) simplify following three variable logic expression

$$Y = \Pi M(1,3,5)$$

$\rightarrow$ $Y = M_1 \cdot M_2 \cdot M_3$

$Y = (A+B+\bar{C})(A+\bar{B}+\bar{C})(\bar{A}+B+\bar{C})$

and further simplify.

Home work

1) write the logic expression in canonical POS form for the truth table.

A	B	C	Y
0	0	0	0
0	0	1	1
1	0	0	0
1	0	1	1
1	1	0	0
1	1	1	0

2) convert the following boolean expression into its standard form.

1) $Y = A\bar{B} + AC + \bar{B}C$

2) $Y = (A+\bar{B}) \cdot (A+C) \cdot (B+\bar{C})$

* Disadvantages of Algebric method of simplification

1) In algebric method of simplification we need to write lengthy equations. find common term use various laws & theorm. so it is time consuming work.

2) top of that sometimes we are not sure whether futher simplification is possible or not. That means whether we have really obtained minimized expression or not.

* karnaugh-Map Simplification :-

This is another simplification technique to reduce the boolean equation.

It overcomes all the disadvantages of the algebric simplification technique.

short form of karnaugh-Map is k-Map and it is graphical method of simplifying a Boolean equation.

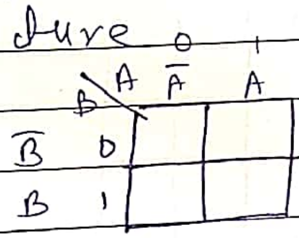
k-Map can be written for 2, 3, 4 --- upto 6 variables.

The k-map is drawn for output Y and input variable (A, B, C -- etc) are used for making the entries in the boxes.

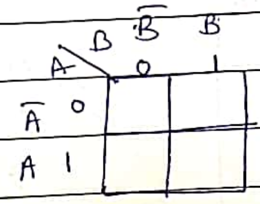
k-Map structure :-

1) 2-variable k-Map :-

structure



or



A & B are inputs or variables.
0 & 1 are value of A or B
Insides 4 boxes are we have to enter the values of Y i.e output

2) ~~3-variable k-Map.~~

2) 3 - variable k-Map :-

A \ BC	$\bar{B}\bar{C}$ 00	$\bar{B}C$ 01	BC 11	$B\bar{C}$ 10
$\bar{A}$ 0				
A 1				

or

C \ AB	$\bar{A}\bar{B}$ 00	$\bar{A}B$ 01	AB 11	$A\bar{B}$ 10
$\bar{C}$ 0				
C 1				

or

BC \ A	$\bar{A}$ 0	A 1
$\bar{B}\bar{C}$ 00		
$\bar{B}C$ 01		
BC 11		
$B\bar{C}$ 10		

A three variable k-map consists of 8 boxes. The positions of variable A, B, C are interchangables.

3) 4 - variable k-Map :-

CD \ AB	$\bar{A}\bar{B}$ 00	$\bar{A}B$ 01	AB 11	$A\bar{B}$ 10
$\bar{C}\bar{D}$ 00				
$\bar{C}D$ 01				
CD 11				
$C\bar{D}$ 10				

or

AB \ CD	$\bar{C}\bar{D}$ 00	$\bar{C}D$ 01	CD 11	$C\bar{D}$ 10
$\bar{A}\bar{B}$ 00				
$\bar{A}B$ 01				
AB 11				
$A\bar{B}$ 10				

A four variable k-map consists of 16 boxes. The position of A, B, C, D is interchangable.

* **Note:-**

The sequence of k-map →

In truth table, the values of inputs follow a standard binary sequence (00, 01, 10, 11) but in k-map the input values are ordered in a different sequence (00, 01, 11, 10). This is as per the gray code and not binary code.

* **K-map Boxes and Associated product terms :-**

The rectangular boxes in K-map are to be filled with value of output Y.

1) 2 - variable K-Map.

A \ B	0 $\bar{B}$	1 B
0 $\bar{A}$	$\bar{A}\bar{B}$ 0	$\bar{A}B$ 1
1 A	$A\bar{B}$ 2	AB 3

2) 3-variable K-Map

A \ BC	00 $\bar{B}\bar{C}$	01 $\bar{B}C$	11 BC	10 $B\bar{C}$
0 $\bar{A}$	$\bar{A}\bar{B}\bar{C}$ 0	$\bar{A}\bar{B}C$ 1	$\bar{A}BC$ 3	$\bar{A}B\bar{C}$ 2
1 A	$A\bar{B}\bar{C}$ 4	$A\bar{B}C$ 5	ABC 7	$AB\bar{C}$ 6

3) 4-variable K-map :-

AB \ CD	$\bar{C}\bar{D}$	$\bar{C}D$	CD	$C\bar{D}$
$\bar{A}\bar{B}$	$\bar{A}\bar{B}\bar{C}\bar{D}$ 0	$\bar{A}\bar{B}\bar{C}D$ 1	$\bar{A}\bar{B}CD$ 3	$\bar{A}\bar{B}C\bar{D}$ 2
$\bar{A}B$	$\bar{A}B\bar{C}\bar{D}$ 4	$\bar{A}B\bar{C}D$ 5	$\bar{A}BCD$ 7	$\bar{A}BC\bar{D}$ 6
AB	$AB\bar{C}\bar{D}$ 12	$AB\bar{C}D$ 13	$ABCD$ 15	$ABC\bar{D}$ 14
$A\bar{B}$	$A\bar{B}\bar{C}\bar{D}$ 8	$A\bar{B}\bar{C}D$ 9	$A\bar{B}CD$ 11	$A\bar{B}C\bar{D}$ 10

* Truth table to k-map :- in SOP form.

example:-

Truth table

A	B	Y
0	0	0
0	1	0
1	0	0
1	1	1

A, B are two input variable
Y → will be output.

⟶

A \ B	0	1
0	0	0
1	0	1

a

B \ A	0	1
0	0	0
1	0	1

* Home work:-

A	B	C	Y
0	0	0	1
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	0
1	1	1	1

write k-Map form above Truth table.

For example :-

1) Represent the equation given below on karnaugh map.

$$Y = \bar{A}\bar{B}\bar{C} + \bar{A}\bar{B}C + A\bar{B}\bar{C} + AB\bar{C} + ABC$$

→ write 1 in output $Y = \bar{A}\bar{B}\bar{C} + \bar{A}\bar{B}C + A\bar{B}\bar{C} + AB\bar{C} + ABC$.

A \ BC	$\bar{B}\bar{C}$	$\bar{B}C$	BC	$B\bar{C}$
$\bar{A}$	1 (0)	1 (1)	0 (3)	0 (2)
A	1 (4)	0 (5)	1 (7)	1 (6)

$\bar{A}\bar{B}\bar{C} = 000 = 0$
$\bar{A}\bar{B}C = 001 = 1$
$A\bar{B}\bar{C} = 100 = 4$
$AB\bar{C} = 110 = 6$
$ABC = 111 = 7$

} Put 1 in this boxes no.

2) Plot the following Boolean expression on K-Map.

$$Y = \bar{A}\bar{B}\bar{C}\bar{D} + \bar{A}\bar{B}CD + \bar{A}BC\bar{D} + ABC̄\bar{D}$$

→

AB \ CD	$\bar{C}\bar{D}$	$\bar{C}D$	CD	$C\bar{D}$
$\bar{A}\bar{B}$	1 (0)	0 (1)	1 (3)	0 (2)
$\bar{A}B$	0 (4)	0 (5)	0 (7)	1 (6)
AB	1 (12)	0 (13)	0 (15)	0 (14)
$A\bar{B}$	0 (8)	0 (9)	0 (11)	0 (10)

* K-map representation of POS form :-

1) POS form equations consist of maxterms.
2) K-map corresponding to POS form equation we have to enter a <u>0</u> corresponding to each maxterm.

1) 2-variable K-map.

A \ B	0 (B)	1 ($\bar{B}$)
0 A	$A+B$	$A+\bar{B}$
1 $\bar{A}$	$\bar{A}+B$	$\bar{A}+\bar{B}$

or

B \ A	A 0	$\bar{A}$ 1
B 0	$A+B$	$\bar{A}+B$
$\bar{B}$ 1	$A+\bar{B}$	$\bar{A}+\bar{B}$

2) 3-variable K-map :-

A \ BC	$B+C$ 00	$B+\bar{C}$ 01	$\bar{B}+\bar{C}$ 11	$\bar{B}+C$ 10
A 0	$A+B+C$	$A+B+\bar{C}$	$A+\bar{B}+\bar{C}$	$A+\bar{B}+C$
$\bar{A}$ 1	$\bar{A}+B+C$	$\bar{A}+B+\bar{C}$	$\bar{A}+\bar{B}+\bar{C}$	$\bar{A}+\bar{B}+C$

3) 4-variable K-map.

AB \ CD	$C+D$ 00	$C+\bar{D}$ 01	$\bar{C}+\bar{D}$ 11	$\bar{C}+D$ 10
$A+B$ 00	$A+B+C+D$	$A+B+C+\bar{D}$	$A+B+\bar{C}+\bar{D}$	$A+B+\bar{C}+D$
$A+\bar{B}$ 01	$A+\bar{B}+C+D$	$A+\bar{B}+C+\bar{D}$	$A+\bar{B}+\bar{C}+\bar{D}$	$A+\bar{B}+\bar{C}+D$
$\bar{A}+\bar{B}$ 11	$\bar{A}+\bar{B}+C+D$	$\bar{A}+\bar{B}+C+D$	$\bar{A}+\bar{B}+\bar{C}+\bar{D}$	$\bar{A}+\bar{B}+\bar{C}+D$
$\bar{A}+B$ 10	$\bar{A}+B+C+D$	$\bar{A}+B+C+\bar{D}$	$\bar{A}+B+\bar{C}+\bar{D}$	$\bar{A}+B+\bar{C}+D$

example :-

1) Represent the following canonical POS expression on K-Map.

$$Y = (A+B+C)(A+\bar{B}+C)(\bar{A}+\bar{B}+C).$$

→

A \ BC	$B+C$	$B+\bar{C}$	$\bar{B}+\bar{C}$	$\bar{B}+C$
A	$A+B+C$	$A+B+\bar{C}$	$A+\bar{B}+\bar{C}$	$A+\bar{B}+C$
$\bar{A}$	$\bar{A}+B+C$	$\bar{A}+B+\bar{C}$	$\bar{A}+\bar{B}+\bar{C}$	$\bar{A}+\bar{B}+C$

→

A \ BC				
	0	1	1	0
	1	1	1	0

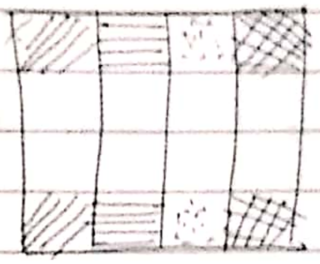
K-map reduction technique for Boolean expression

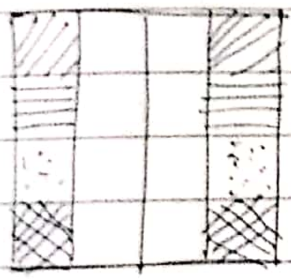
Adjacent cell.

A \ BC	00	01	11	10
0	1	2	3	4
1	5	6	7	8

- 1 and 2 are adjacent
- 1 and 5 are adjacent
- But 1-6 or 2-5 are not adjacent

left most and Right most cells are adjacent





adjacent cells.

Note → The cells on left or right side or at the bottom and top of cells are adjacent cells.
But the cells connected diagonally are not adjacent.

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How to simplification takes place?

1) once the transfer the logic function or truth table on a K-map we have to use the grouping technique for simplifying the logic function.

2) Grouping means combining the terms in the adjacent cells.

3) grouping of either adjacent 1's or adjacent 0's result in simplification of Boolean expression in the SOP or POS forms respectively.

4) group the adjacent 1's then the result of simplification is in SOP form.

5) And if the adjacent 0's are grouped, then the result of simplification in the POS form.

* way of grouping (Pairs, Quads & octets):-

while grouping we should group most number of 1's (or 0's).

The grouping follows binary rule.
i.e we can group 1, 2, 4, 8, 16, 32 ---- number of 1's or 0's. we cannot group 3, 5, 7 --- number of 1's or 0's.

1) Pairs → A group of two adjacent 1's or 0's is called as a pair.

2) Quad → A group of four adjacent 1's or 0's is called as a quad.

3) octet → A group of eight adjacent 1's or 0's is called as octet.

* grouping two adjacent one's -

given k-map :-

A \ BC	$\bar{B}\bar{C}$	$\bar{B}C$	BC	$B\bar{C}$
$\bar{A}$	0	0	1	1
A	0	0	0	0

grouping of adjacent 1's

simplification → $Y = \bar{A}BC + \bar{A}B\bar{C} = \bar{A}B(C+\bar{C})$

$= \bar{A}B$

(or)

A \ BC	$\bar{B}\bar{C}$	$\bar{B}C$	BC	$B\bar{C}$
$\bar{A}$	0	0	1	1
A	0	0	0	0

$Y = \bar{A}B$.

A \ BC	$\bar{B}\bar{C}$	$\bar{B}C$	BC	$B\bar{C}$
$\bar{A}$	0	0	0	0
A	1	0	0	1

$Y = A\bar{C}$.

For example :-

1) For the logical expression given below the draw k-map and obtain the simplified logical expression.

$$Y = \Sigma m(1, 5, 7, 9, 11, 13, 15)$$

Realize the minimized expression using the basic gates.

→ $Y = \Sigma m(1, 5, 7, 9, 11, 13, 15)$

$Y = m_1 + m_5 + m_7 + m_9 + m_{11} + m_{13} + m_{15}$.

Max m_{15} → so we have to draw 4 variable k-map.

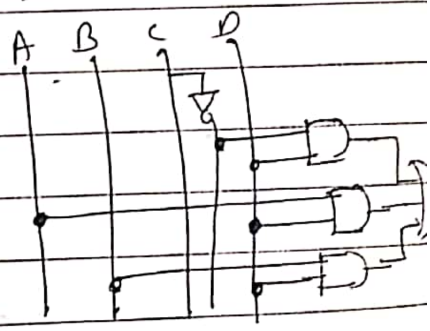
AB \ CD	$\bar{C}\bar{D}$ 00	$\bar{C}D$ 01	CD 11	$C\bar{D}$ 10
$\bar{A}\bar{B}$ 00	0	1	0	0
$\bar{A}B$ 01	0	1	1	0
AB 11	0	1	1	0
$A\bar{B}$ 10	0	1	1	0

BD

AD

$\bar{C}D$

$Y = \bar{C}D + AD + DB$



2) simplify following expression using k-map.

1) $f(A, B, C) = \Sigma m(0, 1, 3, 4, 6)$

2) $f(A, B, C, D) = \Sigma m(0, 1, 2, 4)$

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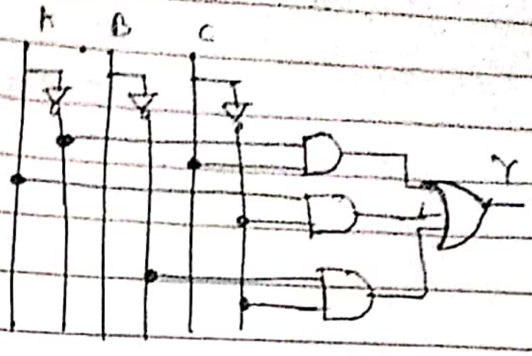
1) $f(A,B,C) = \Sigma m(0,1,3,4,6)$

A \ BC	$\bar{B}\bar{C}$ 00	$\bar{B}C$ 01	BC 11	$B\bar{C}$ 10
$\bar{A}$ 0	1	1	1	0
A 1	1	0	0	1



$Y = \bar{A}C + A\bar{C} + \bar{B}\bar{C}$

2) $f(A,B,C,D) = \Sigma m(0,1,2,4)$

AB \ CD	$\bar{C}\bar{D}$ 00	$\bar{C}D$ 01	CD 11	$C\bar{D}$ 10
$\bar{A}\bar{B}$ 00	1	1	0	1
$\bar{A}B$ 01	1	0	0	0
AB 11	0	0	0	0
$A\bar{B}$ 10	0	0	0	0

$Y = \bar{A}\bar{B}\bar{C} + \bar{A}\bar{B}\bar{D} + \bar{A}\bar{C}\bar{D}$

For Home work / Assignment :-

1) solve the following sop expression with k-map.

1) $f(A,B,C,D) = \Sigma m(0,1,3,4,5,7)$
2) $f(A,B,C) = \Sigma m(0,1,4,5,6,7)$

2) convert the following function into canonical sop form and plot the k-map.

$Y = AB + AC + BC$.

3) solve the following Boolean expression using K-Map
$Y = \Sigma m(m_0, m_1, m_2, m_4, m_{12}, m_{13}, m_{14}, m_{15})$.

* Don't care contitions :-

1) For SOP form we enter 1's corresponding to the combinations of input variables which produce a high output. and we enter 0's. in remaing cells. of K-Map.

2) For POS form we enter's 0's corresponding to the combinations of inputs which produce a low output and enter 1's 1's in remaing cell of K-Map.

3) but it is not always true that cells not containing 1's (in SOP) will contain 0's because some combination of input variable donot occurs.

4) For some function output corresponding to certa combinations of input variable do not matter.

5) In such situation ~~we~~ free to assume a 0 or 1 as output for each of such input combination These contitions are known as the 'Don't care contitions'.
And in the K-map it is represented as X cross mark in the corresponding cell.

For example:-

1) simplify the expression given below using k-map the don't care conditions are indicated by $d(\ )$.

$$Y = \Sigma m(1,3,7,11,15) + d(0,2,5)$$

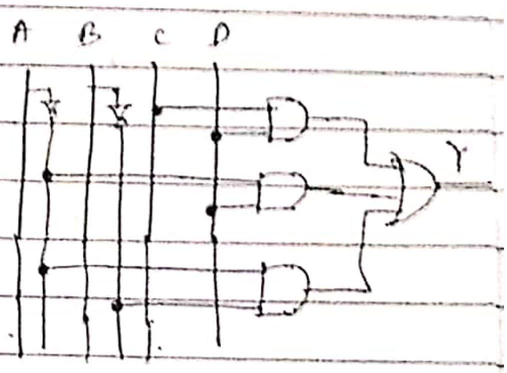
$\Rightarrow$ $Y = m_1 + m_3 + m_7 + m_{11} + m_{15} + d(0,2,5)$.

AB \ CD	$\bar{C}\bar{D}$ 00	$\bar{C}D$ 01	CD 11	$C\bar{D}$ 10
$\bar{A}\bar{B}$ 00	X	1	1	X
$\bar{A}B$ 01	0	X	1	0
AB 11	0	0	1	0
$A\bar{B}$ 10	0	0	1	0

group2 $\rightarrow \bar{A}\bar{B}$

group1 $\rightarrow CD$.

group3 $\rightarrow \bar{A}D$



$$Y = CD + \bar{A}D + \bar{A}\bar{B}$$

2) Minimize the following using k-map.

1) $Y = \Sigma m(0,1,4,5) + \Sigma d(6,7,14,15)$

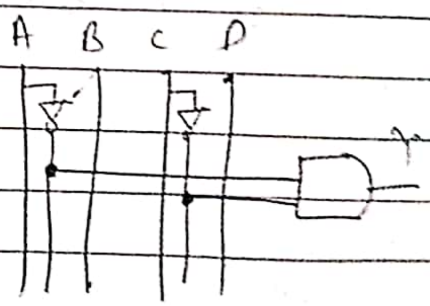
2) $Y = \Sigma m(0,2,4,6) + \Sigma d(12,13,14,15)$.

$\rightarrow$ 1) $Y = \Sigma m(0,1,4,5) + \Sigma d(6,7,14,15)$

AB \ CD	00	01	11	10
00	1	1	0	0
01	1	1	X	X
11	0	0	X	X
10	0	0	0	0

$\bar{A}\bar{C}$

$Y = \bar{A}\bar{C}$

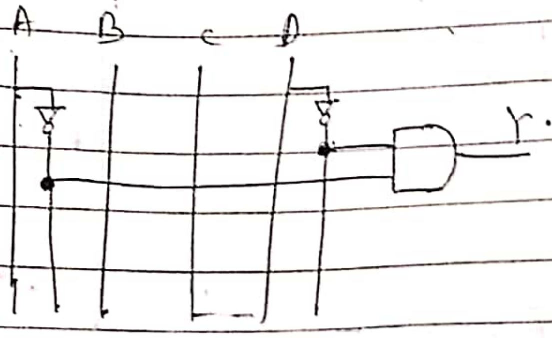


2) $Y = \Sigma m(0,2,4,6) + \Sigma d(12,13,14,15)$

$AB \backslash CD$	$\bar{C}\bar{D}$ 00	$\bar{C}D$ 01	CD 11	$C\bar{D}$ 10
$\bar{A}\bar{B}$ 00	1	0	0	1
$\bar{A}B$ 01	1	0	0	1
AB 11	X	X	X	X
$A\bar{B}$ 10	0	0	0	0

$\bar{A}\bar{D}$

$Y = \bar{A}\bar{D}$



For Assignment:-

1) simplify the following equation using k-map and realize it using logic gates.

$$Y = \Sigma m(0,1,2,3,8,10) + \Sigma d(5,7)$$

2) Minimize the following using k-map.

1) $Y = \Sigma m(0,1,4,5) + \Sigma d(6,7,14,15)$

2) $Y = \Sigma m(0,2,4,6) + \Sigma d(12,13,14,15)$

* simplification of canonical pos from using k-map.
group the 0's

For example :-

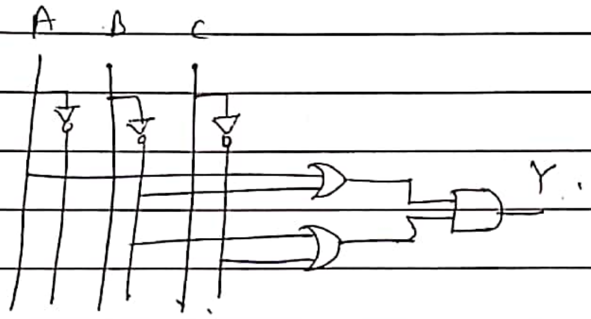
1) Find the expression in the pos form for the k-map.

A \ BC	$B+C$ 00	$B+\bar{C}$ 01	$\bar{B}+\bar{C}$ 11	$\bar{B}+C$ 10
A 0	1	1	0	0
$\bar{A}$ 1	1	1	0	1

$A+\bar{B}$

$\bar{B}+\bar{C}$

$Y = (A+\bar{B})(\bar{B}+\bar{C})$



2) write the simplified expression for output in the pos form for the following expression.

$Y = \Pi M (0,2,3,7)$

3) write the expression for output in pos form for the k-map. shown in fig.

AB \ CD	00	01	11	10
00	0	1	1	0
01	1	1	1	1
11	1	1	1	1
10	0	1	1	0