4.3 Concepts of Minterm and Maxterm :

W-08, W-17

MSBTE Questions

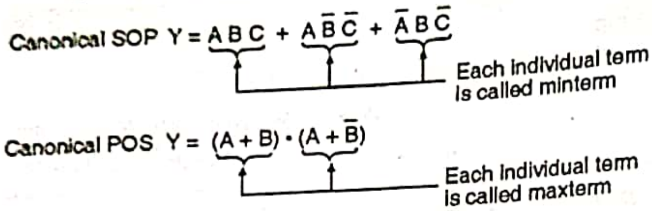
Q. 1 Define minterm and maxterm. Give example. **(W-08, W-17, 2 Marks)**

Minterm :

Each individual term in the canonical SOP form is called as **minterm.** This is shown in Fig. 4.3.1.

Maxterm :

- Each individual term in the canonical POS form is called as **maxterm.** This is shown in Fig. 4.3.1.



(C-1220) **Fig. 4.3.1 : Concept of maxterm and minterm**

- Table 4.3.1 gives the minterms and maxterms for a three variable/literal logic function. Let Y be the output and A, B, C be the inputs.
- The number of minterms and maxterms is $2^3 = 8$. In general for "n" number of variables the number of minterms or maxterms will be 2^n.
- Each minterm is represented by m_i where $i = 0, 1, \ldots, 2^n - 1$ and each maxterm is represented by M_i.

Table 4.3.1 : Minterms and maxterms for three variables

Variables			Minterms	Maxterms
A	B	C	m_i	M_i
0	0	0	$\bar{A}\bar{B}\bar{C} = m_0$	$A + B + C = M_0$
0	0	1	$\bar{A}\bar{B}C = m_1$	$A + B + \bar{C} = M_1$
0	1	0	$\bar{A}B\bar{C} = m_2$	$A + \bar{B} + C = M_2$
0	1	1	$\bar{A}BC = m_3$	$A + \bar{B} + \bar{C} = M_3$
1	0	0	$A\bar{B}\bar{C} = m_4$	$\bar{A} + B + C = M_4$
1	0	1	$A\bar{B}C = m_5$	$\bar{A} + B + \bar{C} = M_5$
1	1	0	$AB\bar{C} = m_6$	$\bar{A} + \bar{B} + C = M_6$
1	1	1	$ABC = m_7$	$\bar{A} + \bar{B} + \bar{C} = M_7$

Ex. 4.3.1 : For the truth table of two variables write the minterms and maxterms.

Soln. :

Refer Table P. 4.3.1 for solution.

Table P. 4.3.1 : Solu...

Variables/literals		Minterms	Maxterms
A	B	m_i	M_i
0	0	$m_0 = \bar{A}\bar{B}$	$M_0 = A + B$
0	1	$m_1 = \bar{A}B$	$M_1 = A + \bar{B}$
1	0	$m_2 = A\bar{B}$	$M_2 = \bar{A} + B$
1	1	$m_3 = AB$	$M_3 = \bar{A} + \bar{B}$

4.3.1 Representation of Logical Expressions using Minterms and Maxterms :

We can represent the logical expression using the minterms and maxterms as follows :

1. $Y = ABC + \bar{A}BC + A\bar{B}\bar{C}$ ← Given logic expression

   $m_7 \quad m_3 \quad m_4$ ← Corresponding minterms (C-6155)

   $\therefore Y = m_7 + m_3 + m_4 = \Sigma m(3, 4, 7)$ ← Other way of representation

   where Σ denotes sum of products.

2. $Y = (A+\bar{B}+C)\,(A+B+C)\,(\bar{A}+\bar{B}+C)$ ← Given expression

   $M_2 \quad M_0 \quad M_6$ ← Corresponding maxterms

   (C-6156)

   $\therefore Y = M_2\ M_0\ M_6$

   $\therefore Y = \Pi M(0, 2, 6)$ ← Other way of representation.

   where Π denotes product of sums.

4.3.2 Writing SOP and POS Forms for a Given Truth Table :

- We know that a logic expression can be represented in the truth table form. For example, the expression $Y = AB + \bar{A}\bar{B}$ which is the Boolean expression for an EX-NOR gate can also be represented using a truth table.
- Hence it is possible to obtain the logic expression in the canonical SOP or POS form if a truth table is given to us.

4.3.3 To Write Canonical SOP Expression for a Given Truth Table :

The procedure to be followed for writing the canonical SOP expression from a given truth table is as follows :

Step 1 : From the given truth table, consider only those combinations of inputs which produce an output Y = 1.

Step 2 : Write down a product term interms of input variables for each such combination.

Step 3 : OR all these product terms produced in step 2 to get the canonical SOP.

Ex. 4.3.2 : From the truth Table P. 4.3.2, obtain the logical expression in the canonical SOP form.

Table P. 4.3.2 : Given truth table

A	B	Y
0	0	0
0	1	1
1	0	1
1	1	0

Soln. :

Step 1 : Consider only the second and the third rows of given truth table corresponding to Y = 1.

Steps 2 and 3 : For the second and the third entries in Table P. 4.3.2(a) write the product terms :

(C-6157) **Table P. 4.3.2(a)**

	A	B	Y	
1	0	0	0	
2	0	1	1	$Y_1 = \bar{A}B$
3	1	0	1	$Y_2 = A\bar{B}$
4	1	1	0	

(Y_1, Y_2 : Boolean expressions in the product forms)

OR (Add) all the product terms to write the final expression in standard SOP form as follows :

$\therefore \quad Y = Y_1 + Y_2 = \bar{A}\,B + A\,\bar{B}$ ...Canonical SOP

$\therefore \quad Y = m_1 + m_2 = \Sigma\, m\,(1, 2)$

Ex. 4.3.3 : For the truth table given in Table P. 4.3.3 write the logic expression in the canonical SOP form.

(C-6412) **Table P. 4.3.3 : Given truth table**

A	B	C	Y	
0	0	0	0	
0	0	1	1	$\bar{A}\bar{B}C\ (m_1)$
0	1	0	0	
0	1	1	0	
1	0	0	1	$A\bar{B}\bar{C}\ (m_4)$
1	0	1	0	
1	1	0	0	
1	1	1	1	$ABC\ (m_7)$

Soln. :

Step 1 : Product terms corresponding to combinations of inputs for which Y = 1.

Step 2 : OR all the product terms to get :

$$Y = \bar{A}\,\bar{B}\,C + A\bar{B}\bar{C} + A\,B\,C$$

$$\therefore \quad Y = m_1 + m_4 + m_7 = \Sigma\, m\,(1, 4, 7)$$

This is the required expression.

4.3.4 To Write a Canonical POS Expression for a Given Truth Table :

Follow the procedure given below to get the expression in canonical POS form.

Step 1 : Consider only those combinations of inputs which produce a low output (Y = 0).

Step 2 : Write the maxterms only for such combinations.

Step 3 : AND these maxterms to obtain the expression in canonical POS form.

Ex. 4.3.4 : Write the logic expression in canonical POS form for the truth table shown in Table P. 4.3.4.

(C-6413) **Table P. 4.3.4 : Given truth table**

A	B	C	Y	
0	0	0	0	$A + B + C\ (M_0)$
0	0	1	1	
0	1	0	1	
0	1	1	0	$A + \bar{B} + \bar{C}\ (M_3)$
1	0	0	1	
1	0	1	0	$\bar{A} + B + \bar{C}\ (M_5)$
1	1	0	0	$\bar{A} + \bar{B} + C\ (M_6)$
1	1	1	1	

Soln. :

Step 1 : Write maxterms for the combinations of input which produce Y = 0.

Step 2 : AND (take product of) all the maxterms to get canonical POS form :

- ANDing (taking product of) all the maxterms written in step 1, we get,

$$Y = (A + B + C) \cdot (A + \bar{B} + \bar{C})\,(\bar{A} + B + \bar{C})\,(\bar{A} + \bar{B} + C)$$

- This is the required logic expression in the canonical POS form.
- This expression can also be written as,

$$Y = M_0 \cdot M_3 \cdot M_5 \cdot M_6$$

$$\text{or} \quad Y = \Pi\,(0, 3, 5, 6)$$

4.3.5 Conversion from SOP to POS and Vice Versa :

- It is important to note that the SOP and POS forms written for the same truth table are always logically equivalent.
- This point can be proved by solving the following example.

Ex. 4.3.5 : For the given truth table write the logical expressions in the canonical SOP and POS forms and prove their equivalence.

Step 2 : Simplify this SOP exp... terms for common factors.

Ex. 4.4.1 : Simplify the following three variable Boolean expression :

$Y = \Sigma m (2, 4, 6)$

Soln. : The given expression can be written in SOP form as follows :

$$Y = m_2 + m_4 + m_6$$

$$= \bar{A} B \bar{C} + A \bar{B} \bar{C} + A B \bar{C}$$

$$= \bar{A} B \bar{C} + A\bar{C} (\bar{B} + B)$$

$$\text{But } \bar{B} + B = 1$$

$$\therefore Y = \bar{A} B \bar{C} + A\bar{C}$$

$$\therefore Y = \bar{C} (\bar{A} B + A)$$

$$\text{But } A + BC = (A + B)(A + C)$$

$$\text{hence } (A + \bar{A} B) = (A + \bar{A})(A + B)$$

$$\therefore Y = \bar{C} (A + \bar{A})(A + B) = \bar{C} (1) (A + B)$$

$$\therefore Y = \bar{C} (A + B) \quad \text{...Ans.}$$

This is simplified expression.