Grade 11 Chapter 3 Number patterns - Day 1

1. Quadratic relationship $T_n = an^2 + bn + c$

- The 2nd difference is constant = 2a
- Term 1 of the 1st difference = 3a + b
- Term 1 of the sequence = a + b + c

Example

Determine the general term of the sequence: 5; 6; 13; 26

Solution

$\boldsymbol{a+b+c}$	**5**		**6**		**13**		**26**
$\boldsymbol{3a+b}$		**1**		**7**		**13**	
$\boldsymbol{2a}$			**6**		**6**		

$2a = 6$
$a = 3 \ldots\ldots (1)$

$3a + b = 1$ sub in (1)
$3(3) + b = 1$
$b = -8 \ldots\ldots (2)$

$a + b + c = 5$ sub in (1) and (2)
$3 - 8 + c = 5$
$c = 10$

$\therefore T_n = 3n^2 - 8n + 10$

Example

$a+b+c = 6$	;		17	;	34	;	57
$3a+b = 11$				17		23	
$2a = 6$					6		

$T_n = an^2 + bn + c$

$2a = 6$
$a = 3$

$3a + b = 11$
$3(3) + b = 11$
$b = 2$

$a + b + c = 6$
$3 + 2 + c = 6$
$c = 1$

$\therefore T_n = 3n^2 + 2n + 1$

Example 3:

The sequence 18; 16; 12; 6; -2; … is quadratic.

(a) Determine a formula for T_n.

(b) Determine the value of T_{15}.

(c) Which term of the sequence is equal to -72?

Solution:

(a)

$$a+b+c = 18 \quad 16 \quad 12 \quad 6 \quad -2$$

$$3a+b = -2 \quad -4 \quad -6 \quad -8$$

$$2a = -2 \quad -2 \quad -2$$

$2a=-2 \qquad \therefore a=-1$

$3a+b=-2$

$\therefore 3(-1)+b=-2 \qquad \therefore b=1$

$a+b+c=4$

$\therefore (-1)+1+c=18 \qquad \therefore c=18$

$T_n = an^2+bn+c$

$\therefore T_n = -n^2+n+18$

(b) $T_{15} = -(15)^2+15+18 = -192$

(c) $-n^2+n+18=-72$

$\therefore n^2-n-90=0$

$\therefore (n-10)(n+9)=0$

$\therefore n=10$ of $n=-9$

N.V.T.

$\therefore n=10$

Homework:

Exercise 1 p 45 no f, g

(f) Consider the quadratic pattern: 3; -1; 2; …

(1) Determine the general term (T_n).

(2) Which term of the pattern is equal to 1839?

(g) Given the sequence 1; -5; -13; -23; …

(1) Find the general term.

(2) Which term of the pattern is equal to -643?

Exercise 2 p 47 no c, e

(c) The following number pattern is quadratic: -4; -1; -2; -7; …

(1) Determine the general term of the sequence of first differences.

(2) Calculate the difference between the 25th and 26th terms of the quadratic pattern.

(3) Determine the general term of the quadratic number pattern.

(4) Explain why this quadratic number pattern will never have a positive term.

(e) Given the quadratic sequence: 846; 789; 734; 681; …

Determine the position and the value of the term with the lowest value.

Extra challenges:

3.2 A linear pattern has a difference of 3 between consecutive terms and its 20^{th} term is equal to 64 (that is $T_{20} = 64$).

3.2.1 Determine the value of T_{22}. (1)

3.2.2 Which term in the pattern will be equal to $3T_5 - 2$? (4)

3.3 Consider the quadratic pattern: $5 ; 12 ; 29 ; 56 ; \ldots$

3.3.1 Write down the NEXT TWO terms of the pattern. (2)

3.3.2 Prove that the first differences of this pattern will always be odd. (3)

QUESTION 3

Consider the quadratic pattern: $-9; -6 ; 1 ; 12 ; x ; \ldots$

3.1 Determine the value of x. (1)

3.2 Determine a formula for the n^{th} term of the pattern. (4)

3.3 A new pattern, P_n, is formed by adding 3 to each term in the given quadratic pattern. Write down the general term of P_n in the form $P_n = an^2 + bn + c$. (1)

3.4 Which term of the sequence found in QUESTION 3.3 has a value of 400? (4)
[10]

QUESTION 4

Consider the following quadratic number pattern: $-7 ; 0 ; 9 ; 20 ; \ldots$

4.1 Show that the general term of the quadratic number pattern is given by $T_n = n^2 + 4n - 12$. (4)

4.2 Which term of the quadratic pattern is equal to 128? (4)

4.3 Determine the general term of the first differences. (3)

4.4 Between which TWO terms of the quadratic pattern will the first difference be 599? (3)
[14]

Memo van ekstra uitdagende somme

3.2.1	$T_{22} = 64+3+3$ $= 70$	✓ answer/*antwoord* (1)
3.2.2	$T_{20} = 3n+b$ $64 = 3(20)+b$ $b = 4$ $T_n = 3n+4$ $3T_5 - 2 = 3[3(5)+4]-2$ $= 55$ $3n+4 = 55$ $3n = 51$ $n = 17$	✓ $64 = 3(20)+b$ ✓ $3[3(5)+4]-2$ ✓ $3n+4 = 55$ ✓ $n = 17$ (4)
3.3.1	5 12 29 56 7 17 27 10 10 93 ; 140	✓ 93 ✓ 140 (2)
3.3.2	7 17 27 10 10 $T_n = 10n-3$ $= (10n-2)-1$ $= 2(5n-1)-1$ Since $10n-2 = 2(5n-1)$, $10n-2$ is even for any value of n. Thus T_n is always odd, since for any value of n, T_n is always one less than an even number *Aangesien* $10n-2 = 2(5n-1)$, *is* $10n-2$ *dus ewe vir enige waarde van n.* *Dus* T_n *is altyd onewe, want vir enige waarde van n, is* T_n *altyd een minder as 'n ewe getal*	✓ $T_n = 10n-3$ ✓ $10n-2 = 2(5n-1)$ ✓explanation/*verduidlkng* (3)

QUESTION/*VRAAG* 3

3.1	$-9 \quad 6 \quad 1 \quad 12 \quad x$ $3 \quad 7 \quad 11 \quad x\text{-}12$ $4 \quad 4 \quad x\text{-}23$ $x-23=4$ $x=27$	✓answer/*antw.* (1)
3.2	$2a=4$ $a=2$ $3a+b=3$ $6+b=3$ $b=-3$ $a+b+c=-9$ $2-3+c=-9$ $c=-8$ $T_n=2n^2-3n-8$	✓ $a=2$ ✓ $b=-3$ ✓ $c=-8$ ✓ $T_n=2n^2-3n-8$ (4)
3.3	$T_y=2n^2-3n-8+3$ $=2n^2-3n-5$ CA from 3.2	✓answer/*antw.* (1)
3.4	$T_n=400$ $2n^2-3n-5=400$ $2n^2-3n-405=0$ $(n-15)(2n+27)=0$ $n=15$ or $n \neq \frac{-27}{2}$ CA from 3.3	✓ equating/*verg.* ✓std form/*stand vorm* ✓factorisation/*fakt.* ✓ $n=15$

VRAAG 4

4.1	$-7 \quad 0 \quad 9 \quad 20$ $7 \quad 9 \quad 11$ $2 \quad 2$ $2a = 2$ $a = 1$ $3(1) + b = 7$ $b = 4$ $(1) + (4) + c = -7$ $c = -12$ $\therefore T_n = n^2 + 4n - 12$	✓ $2a = 2$ ✓ a waarde ✓ b waarde ✓ c waarde (4)
4.2	$n^2 + 4n - 12 = 128$ $n^2 + 4n - 140 = 0$ $(n+14)(n-10) = 0$ $n \neq -14$ or $n = 10$ ongeldig $\therefore n = 10$	✓ vergelyking ✓faktore ✓antwoords vir n ✓$n = 10$ (keuse) (4)
4.3	$-7 ; 0 ; 9 ; 20 ; \ldots$ eerste verskil 7 9 11 tweede verskil 2 2 $F_n = 2n + c$ $F_1 = 2(1) + c = 7$ $\therefore c = 5$ $F_n = 2n + 5$	✓eerste verskil Slegs antwoord: Vol Punte ✓$c = 5$ (3)
4.4	$F_n = 2n + 5 = 599$ $2n = 594$ $\therefore n = 297$ hierdie verskil sal tussen term 297 term 298 wees	✓ stel gelyk ✓297 ✓298(3) **[14]**