ASTRONOMICAL SCALES

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1 Astronomical Distance, Distance, Mass and Time Scales:

In astronomy, we are interested in measuring various physical quantities, such as mass, distance, radius, brightness and luminosity of celestial objects. The radius, R_E,and mass, M_E, of the Earth are

$$R_E = 6378Km$$
$$M_E = 5.974 \times 10^{24}Kg$$

In comparison, the Sun is about a million times more massive with about 100 times larger radius:

$$R_S = 6.96 \times 10^5 Km = R_\Theta$$
$$M_S = 1.989 \times 10^{30} Kg = M_\Theta$$

Astronomical Distance:

The mean distance between the Sun and the Earth is called one **astronomical unit** $(AU = 1.496 \times 10^{11}m.$ Distances in the solar system are measured in this unit.

Another unit is the **light year** (ly), used for measuring distances to stars and galaxies. It is defined as the the distance travelled by light in one year. $1ly = 9.460 \times 10^{15}m = 6.323 \times 10^4 AU$

The **parsec**(pc) is a third unit of length measurement in astronomy. It is defined as the distance at which the radius of Earth's orbit subtends an angle of $1''$. $1pc = 3.262ly = 2.062 \times 10^5 AU = 3.085 \times 10^{16}m$

Dimensions of Astronomical Objects:

The sizes of stars or stellar dimensions are usually measured in units of solar radius (R_Θ). For example, Sirius the brightest star in the sky, has radius $2R_\Theta$. The radius of the star Aldebaran in Taurus is $40R_\Theta$ and that of Antares in Scorpius is $700R_\Theta$

Mass:

Stellar masses are usually measured in units of solar mass M_Θ. For example, the mass of our galaxy is $\sim 10^{11}M_\Theta$. The mass of a globular cluster is of the order of $10^5 - 10^6 M_\Theta$. S. Chandrasekhar showed that the mass of a white dwarf star cannot exceed $1.4M_\Theta$. This is called the Chandrasekhar limit.

Time Scales

The present age of the Sun is about 5 billion years. It has been estimated that it would live for another 5 billion years in its present form. The age of our galaxy may bearound 10 billion years. Various estimates of the age of the universe itself give a figure between 12 and 16 billion years. On the other hand, if the pressure inside a star is insufficient to support it against gravity, then it may collapse in a time, which may be measured in seconds, rather than in millions of years.

2 Brightness, radiant Flux and Luminosity:

A star might look bright because it is closer to us. And a really brighter star might appear faint because it is too far. We can estimate the apparent brightness of astronomical objects easily, but, if we want to measure their **real or intrinsic brightness**, we must take their distance into account. The **apparent brightness** of a star is defined in terms of the **apparent magnitude** of a star.

Apparent Magnitude:

Apparent magnitude of an astronomical object is a measure of how bright it appears. According to the magnitude scale, a smaller magnitude means a brighter star.

The flux density F received at Earth from a star can be expressed in terms of the apparent magnitude, m, defined as

$$m = -2.5 \log_{10} \frac{F}{F_0}$$

where F_0 is a reference flux density. The constant 2.5 is used for historical reasons. For two stars with flux densities F_1 and F_2,

$$m_1 - m_2 = -2.5 \log_{10} \frac{F_1}{F_2}$$

or

$$\frac{F_2}{F_1} = 100^{(m_1 - m_2)/5}$$

The star with larger m appears less bright. If $m_1 - m_2 = 5$, then $F_2 = 100F_1$

The magnitude system is designed to specify the flux of an object with respect to the flux of some standard source used as a reference. This is convenient because measurement of relative flux is often much easier and more reliable in comparison with the absolute flux. Many instrumental errors as well as distortions due to atmosphere cancel out when measuring the ratio of two fluxes but will lead to errors in measurement of an absolute flux. The magnitude of the reference source is assigned some fixed value, such as zero.

The flux and hence the observed magnitude depend on the instrument, which may have different sensitivities at different wavelengths. Hence, what is measured is

$$F = \int_0^\infty S_\nu F_\nu d\nu$$

where S_ν is the sensitivity function of the instrument and ν is the frequency.

The Greek astronomer Hipparchus was the first astronomer to catalogue stars visible to the naked eye. He divided stars into six classes, or apparent magnitudes, by their relative brightness as seen from Earth. He numbered the apparent magnitude (m) of a star on a scale of 1 (the brightest) to 6 (the least bright). This is the scale on which the apparent brightness of stars, planets and other objects is expressed as they appear from the Earth. The brightest stars are assigned the first magnitude $(m = 1)$ and the faintest stars visible to the naked eye are assigned the sixth magnitude $(m = 6)$.

The magnitude scale is actually a non-linear scale. The response of the eye to increasing brightness is nearly logarithmic. We, therefore, need to define a logarithmic scale for magnitudes in which a difference of 5 magnitudes is equal to a factor of 100 in brightness. On this scale, the brightness ratio corresponding to 1 magnitude difference is $100^{1/5}$ or 2.512. Therefore, a star of magnitude 1 is 2.512 times brighter than a star of magnitude 2.

The apparent magnitude and brightness of a star do not give us any idea of the total energy emitted per second by the star. This is obtained from **radiant flux** and the **luminosity** of a star. The **luminosity** of a body is defined as the total energy radiated by it per unit time. **Radiant flux** at a given point is the total amount of energy flowing through per unit time per unit area of a surface oriented normal to the direction of propagation of radiation. The unit of radiant flux is $erg.s^{-1}cm^{-2}$ and that of luminosity is $erg.s^{-1}$.In astronomy, it is common to use the cgs system of units.

The radiated energy includes all wavelengths. The radiant flux of a source depends on two factors:

1. the radiant energy emitted by it, and
2. the distance of the source from the point of observation.

Suppose a star is at a distance r from us. Let us draw an imaginary sphere of radius r round the star. The surface area of this sphere is $4\pi r^2$. Then the radiant flux F of the star, is related to its luminosity L as follows:

$$F = \frac{L}{4\pi r^2}$$

The luminosity of a stellar object is a measure of the intrinsic brightness of a star. It is expressed generally in the units of the solar luminosity, L_Θ, where $L_\Theta = 4 \times 10^6 W = 4 \times 10^{33} erg.s^{-1}$. For example, the luminosity of our galaxy is about $10^{11} L_\Theta$.

Now, the energy from a source received at any place, determines the brightness of the source. This implies that F is related to the brightness b of the source: the brighter the source, the larger would be the radiant flux at a place. Thus, the flux received at a place also depends on its distance from the source. Therefore, two stars of the same apparent magnitude may not be equally luminous, as they may be located at different distances from the observer: A star's apparent brightness does not tell us anything about the luminosity of the star. We need a measure of the **true or intrinsic brightness** of a star.

Absolute Magnitude:

The apparent magnitude is related to the flux density of an astronomical object. We also need a measure that quantifies the luminosity or intrinsic brightness of an object. This is provided by the absolute magnitude, denoted by M. It is defined as the apparent magnitude of an object when it is placed at a distance of 10 pc from the observer. For a source that radiates isotropically, the flux density $F(r)$ in vacuum is proportional to $1/r^2$, where r is the distance of the observer from the source. Hence,

$$\frac{F(r)}{F(10)} = \left(\frac{10pc}{r}\right)^2$$

where $F(10)$ is the flux density at a distance of 10 pc. This implies that

$$m - M = -2.5 \log_{10} \frac{F(r)}{F(10)} = -2.5 \, log_{10} \left(\frac{10pc}{r}\right)^2 = 5 \log_{10} \frac{r}{10pc} = 5 \log_{10} r - 5$$

.The difference $m - M$ is called the distance modulus because it is a measure of the distance of the object.

The measurement of absolute magnitude is clearly much more complicated in comparison to that of apparent magnitude. It requires knowledge of the distance from the source. A direct measurement of distance is possible only using parallax. This can be used only for nearby sources. For larger distances, the luminosity is deduced indirectly. One often uses the fact that the luminosity of many sources shows some definite relationship to some other observables. In some cases, the luminosity is approximately constant for all the sources belonging to a particular class. Hence one is able to indirectly deduce the absolute magnitude of these sources..

3 Measurement of Astronomical Quantities:

The brightness of heavenly objects depends on their distances from us, the measurement of distance is very important in astronomy.

Astronomical distance:

The parallax method is the only direct method for measuring astronomical distances. All other methods rely on some physical property of a star, through which we can deduce its luminosity. Hence the extracted distance may depend on assumptions about its physical properties.

Parallax is the apparent change in the position of an object due to a change in the location of the observer. If we observe an object from different positions, as illustrated in Figure 1, we see it in different directions. All stars and galaxies are located at very large distances from Earth in comparison to typical distances within the solar system. Due to this large distance, the parallax observable for a star is very small. The largest parallax arises due to the revolution of Earth. Even this is very small and can be detected only by very precise measurements. Furthermore, this measurement is possible only for stars close to us. The shift is undetectable for stars that are very far away.

Let r be the distance of a star and L the distance moved by the observer perpendicular to the line joining the star to the observer, as shown in Figure 1. Then the parallax is the angular shift $\delta\theta$,

$$\delta\theta = \frac{L}{r}$$

This relationship is valid for small angles. The distance L is also called the **baseline**. By measuring $\delta\theta$, we can estimate the distance of the star.

In order to measure astronomical distances, we need large baselines. Even for measuring the distance to the nearest star, we require a baseline length greater than the Earth's diameter. This is because the distance of the star is so large that the angle measured from two diametrically opposite points on the Earth will differ by an amount which cannot be measured. Therefore, we take the diameter of the Earth's orbit as the baseline, and make two observations at an interval of six months.

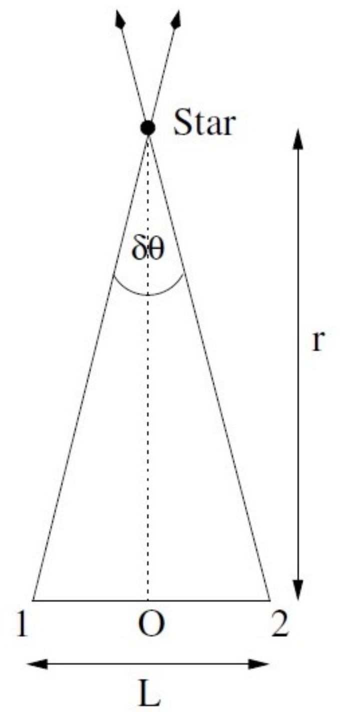

Figure 1: Parallax angle and base line

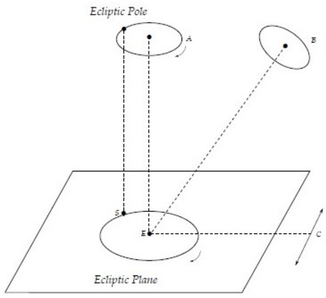

Figure 2: The Sun (S), as observed from Earth (E), appears to move in a circle in the ecliptic plane. A star A which lies directly above the Sun along the line perpendicular to the ecliptic plane also appears to move in a circle. A star B at any other position appears to move in an ellipse. A star C which lies in the ecliptic plane, shows a straight line motion.

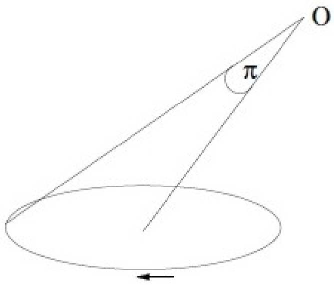

Figure 3: Due to the annual motion of the Earth, the stars appear to move in an ellipse. The parallax π is the angle subtended by the semi-major axis of this ellipse at the position of the observer O.

Due to the annual motion of Earth, a star appears to move in an elliptical orbit. The eccentricity of the ellipse is determined by the angular position of the star, as shown in Figure 2. The angle subtended by the semi-major axis of this ellipse is called the **annual parallax**, π (Figure). It is the same as the angle subtended by the orbital radius of Earth (1 AU) at the star.

One half of the maximum change in angular position of the star is defined as its annual parallax. From Fig. 1, the distance r of the star is given by

$$\frac{d_{SE}}{r} = \tan\theta$$

where d_{SE} is the average distance between the Sun and the Earth. Since the angle θ is very small, $\tan\theta \approx \theta$, and we can write

$$r = \frac{d_{SE}}{\theta}$$

Since, $d_{SE} = 1AU$, we have

$$r = \frac{1AU}{\theta}$$

Since $1 radian = 206,265''$, this implies that if the angle subtended $\delta\theta = 1''$, then the distance

$$r = \frac{1AU}{\theta} = 206,265AU = 1pc$$

Thus, if we measure θ in arc seconds, then the distance is said to be in **parsecs**. Hence, one parsec is the distance of an object that has a parallax of one second of an arc $(1'')$. The nearest star Proxima Centauri has a parallax angle $0.77''$. Thus its distance is $1.3pc$. Since the distance is proportional to $1/\theta$, the more distant a star is, the smaller is its parallax.

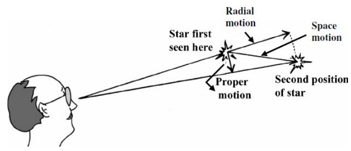

Figure 4: Radial and proper motion of a star

The stars themselves are in motion with respect to the Sun. Therefore, the observed parallax of a star also depends on its **proper motion**, μ. The distance as well as μ are both extracted simultaneously by parallax measurements. The proper motion of all stars is observed to be approximately independent of time. Hence this gives a contribution to parallax that increases linearly with time (t) in a fixed direction, that is, $\delta\theta = \mu t$. In contrast, the star appears to move in an ellipse due to the annual motion of the Earth. Hence this contribution does not show a linear dependence on time and also changes direction. In particular, it becomes zero after one complete year. Due to the different time dependences of these two components, they can be separated by making a large number of measurements at different times. Hence we can extract both the proper motion and the distance of a star.

The motion of a star can be resolved along two directions:

1. Motion along the line of sight of the observer, (either towards or away from the observer) is called the radial motion.

2. Motion perpendicular to the line of sight of the observer is called proper motion (Fig. 4)

Radial motion causes the spectral lines of a star to shift towards red (if the motion is away from the observer) or towards blue (if the motion is towards the observer). This shift is the well-known **Doppler shift**. The proper motion is very slow. It is measured over an interval of 20 to 30 years. It is expressed in arc seconds per year. The average proper motion for all naked eye stars is less than $0.1 arc.second/yr$. The proper motion is denoted by μ. For a star at a distance r from the Earth it is related to its transverse velocity as follows :

$$\textit{Proper motion} = \frac{\textit{transverse velocity}}{\textit{distance of the Star}}$$

or

$$\mu = \frac{v_\theta}{r}$$

Hence, $v_\theta = \mu r$

If μ is measured in units of arc-seconds per year and r in pc, the transverse velocity is given by

$$v_\theta(kms^{-1}) = 4.74\mu r$$

If we add the radial velocity vector and the proper motion vector, we obtain the space velocity of a star.

Stellar Radii:

There are several ways of measuring the radii of stars. We will mainly give emphasis to following two methods:

- the direct method, and
- the indirect method.

Direct Method

We use this method to measure the radius of an object that is in the form of a disc. In this method, we measure the angular diameter and the distance of the object from the place of observation

If $\theta(rad)$ is the angular diameter and r is the distance of the object from the observer then the diameter of the stellar object will be

$$D = \theta \times r$$

This method is useful for determining the radii of the Sun, the planets and their satellites. Since stars are so far that they cannot be seen as discs even with the largest telescopes, this method cannot be used to find their radii. For this we use other methods.

The luminosity of a star can also reveal its size since it depends on the surface area and temperature of star. This provides a basis for the indirect method of determining stellar radii.

Indirect methods:

To obtain stellar radii, we can also use Stefan-Boltzmann law of radiation

$$F = \sigma T^4$$

where F is the radiant flux from the surface of the object, σ, Stefan's constant and T, the surface temperature of the star. We know that the luminosity L of a star is defined as the total energy radiated by the star per second. Since $4\pi R^2$ is the surface area, we can write

$$L = 4\pi R^2 F$$

where R is the radius of the star. If the star's surface temperature is T, using the value of F we obtain

$$L = 4\pi R^2 \sigma T^4$$

The knowledge of L and T gives the value of R.

Now let us consider two stars of radii R_1 and R_2 and surface temperatures T_1 and T_2, respectively. The ratio of luminosities of these two stars will be

$$\frac{L_1}{L_2} = \left(\frac{R_1}{R_2}\right)^2 \left(\frac{T_1}{T_2}\right)^4$$

Masses of Stars Scales

Two stars revolving around each other form a binary system. Fortunately, a large fraction of stars are in binary systems and therefore their masses can be determined.

Now suppose M_1 and M_2 are the masses of the two stars and a is the distance between them, then we can write Kepler's third law as

$$\frac{GP^2}{4\pi^2}(M_1 + M_2) = a^3$$

where P is the period of the binary system and G is the constant of gravitation. This relation gives us the combined mass of the two stars. However, if the motion of both the stars around the common centre of mass can be observed, then we have

$$M_1 a_1 = M_2 a_2$$

where a_1 and a_2 are distances from the centre of mass. Then both these equations allow us to estimate the masses of both the stars. Masses of stars are expressed in units of the solar mass, $M_\Theta = 2 \times 10^{30} kg$. Most stars have masses between $0.1 M_\Theta$ and $10 M_\Theta$..

Stellar Temperature:

The temperature of a star can be determined by looking at its spectrum or colour. The radiant flux (F_λ) at various wavelengths (λ) is quite similar to the one obtained for a black body at a certain temperature. Assuming the star to be radiating as a black body, it is possible to fit in a Planck's curve to the observed data at temperature T. This temperature determines the colour of the star.

The temperature of a star (corresponding to a black body) may be estimated using Wien's law:

$$\lambda_{max} T = 0.29 cmK$$

Such a temperature is termed as surface temperature, T_s. In general it is difficult to define the temperature of a star. For instance the temperature obtained from line emission is indicative of temperature from a region of a star where these lines are formed. Similarly the effective temperature of a star corresponds to the one obtained using Stefan-Boltzman law, i.e.,

$$F = \sigma T_e^4$$

The effective temperature T_e is defined as the temperature of a blackbody that radiates the same total flux as the star. Consider a star of radius R located at a distance r from Earth. The flux density at the surface of the star is

$$F_S = \sigma T_e^4$$

, which implies that the luminosity of the star is $L = 4\pi R^2 F_S$. The flux density F_E at Earth is given by

$$F_E = \frac{L}{4\pi r^2} = \frac{R^2}{r^2} F_S = \frac{\sigma \delta^2 T_e^4}{4}$$

where $\delta = \frac{2R}{r}$ is its angular diameter. Hence an observation of the flux density F_E and the angular diameter gives an estimate of T_e. We can find various empirical relationships among different stellar parameters, e.g., mass, radius, luminosity, effective temperatures, etc. Observations show that the luminosity of stars depends on their mass. We find that the larger the mass of a star, the more luminous it is. For most stars, the mass and luminosity are related as

$$\frac{L}{L_\Theta} = \left(\frac{M}{M_\Theta} \right)^{3.5}$$

TRY TO SOLVE:

1. The apparent magnitude of full moon is -12.5 and that of Venus at its brightest is -4.0. Which is brighter and by how much?

2 The apparent magnitude of the Sun is -26.8. Find its absolute magnitude.

3. The mass of star Sirius is thrice that of the Sun. Find the ratio of their luminosities and the difference in their absolute magnitudes. Taking the absolute magnitude of the Sun as 5, find the absolute magnitude of Sirius.